Appendix A: Entropy calculation

From the form of matrix (69) we conclude that a block diagonal with a single non trivial 2×2 block of the form

$$\rho_{\rm inR} = \mathbf{D} \otimes 0, \tag{A1}$$

where

$$\mathbf{D} \equiv \frac{1}{2} \left|\Psi_+(t)\right\rangle \left\langle\Psi_+(t)\right| + \frac{1}{2} \left|\Psi_-(t)\right\rangle \left\langle\Psi_-(t)\right| . \tag{A2}$$

One can compute the eigenvalues of the Hermitian matrix $\mathbf{D}$ (whose matrix elements can be found for instance via the Gram-Schmidt algorithm) and find that they are given by $(1 \pm |d|)/2$ with

$$d \equiv \int db\, \overline{\psi}(b)\psi(b) e^{i[E_+(b)-E_-(b)]t}. \tag{A3}$$

Thus the entropy is

$$S = -\frac{(1+|d|)}{2} \log\left[\frac{(1+|d|)}{2}\right] - \frac{(1-|d|)}{2} \log\left[\frac{(1-|d|)}{2}\right]. \tag{A4}$$

The previous expression can be put in the form of (71) via the substitution $\delta = 1 - |d|$, seen in equation (72).

Appendix B: Bound states and oscillating universes

Assuming $\mu > 0$ and taking $s = -1$ then equation (40) has a solution with negative energy (a bound state) or a cosmological constant solution with energy

$$E_{s,\mu}(b) = -\frac{3V_0}{8\pi G\gamma^2} \frac{1}{\Delta_s \ell_p^2} \left(\sinh(\sqrt{\Delta_s}\ell_p b_\mu)\right)^2 \tag{B1}$$

with b_μ satisfying the following equation

$$\frac{3}{8\pi\gamma^2} \frac{\sinh(2\sqrt{\Delta_s} b_\mu \ell_p)}{2\sqrt{\Delta_s}} = \mu. \tag{B2}$$

The wave function for such a solution is

$$\Psi_{\rm BU}(\nu) = \mathscr{N} \exp(-|\nu| b_\mu), \tag{B3}$$

where $\mathscr{N}$ is a normalization factor. Notice that the amplitude decays exponentially for large volumes. This is a universe of Planckian size to which the name ?baby universe? would seem to apply.

Appendix C: An alternative definition of coarse-graining entropy

There is a natural alternative notion of entropy associated with the situation where the set of states that are seen as equivalent by the observer can be organized in 'bins' and mathematically defined by a set of projection operators P_i. From a given density matrix ρ, one defines the coarse-grained density matrix [**?** **?**]

$$\rho^{\rm CG} \equiv \sum_i \frac{{\rm Tr}[P_i\rho]}{{\rm Tr}[{\rm P_i}]} P_i, \tag{C1}$$

which satisfies all the properties of a density matrix, and gives equal probability to all the elements in each given bin of equivalent states. It flattens the probability distribution in each set of equivalent states representing the complete ignorance of the coarse-grained observer concerning the difference of such states. The coarse-grained entropy is defined as

$$S(\rho^{\rm CG}) \equiv -{\rm Tr}[\rho^{\rm CG} \log(\rho^{\rm CG})]. \tag{C2}$$

It follows from standard results that

$$S(\rho^{\rm CG}) \geq S(\rho) \equiv -{\rm Tr}[\rho \log(\rho)]. \tag{C3}$$

This definition can of course be used in both the cases studied in Section II E and Section III B (see below).

1. Coarse-graining entropy in the situation of Section II E

We now want to define the coarse-grained density matrix. In our simple case, we had only two projectors encoding the equivalence under our coarse-graining of 1 and 2 states. Namely, in terms of asymptotic states used to write the previous finite dimensional (4) matrix, the projectors are

$$P_1 \equiv |b_0, 1\rangle \langle b_0, 1| + |b_0, 2\rangle \langle b_0, 2| = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{C4}$$

and

$$P_2 = \mathbf{1} - P_1 \equiv |-b_0, 1\rangle \langle -b_0, 1| + |-b_0, 2\rangle \langle -b_0, 2| = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{C5}$$

From the definition of the coarse-grained density matrix (C1), we obtain

$$\boldsymbol{\rho}_{in}^{CG} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \to \quad \boldsymbol{\rho}_{out}^{CG} = \frac{1}{4} \begin{pmatrix} 1 + |B(b_0)|^2 & 0 & 0 & 0 \\ 0 & |A(-b_0)|^2 & 0 & 0 \\ 0 & 0 & 1 + |B(b_0)|^2 & 0 \\ 0 & 0 & 0 & |A(-b_0)|^2 \end{pmatrix}.$$

One can now straightforwardly compute the coarse-graining entropy (C2). We observe that it is constantly equal to log(2) before the big bang and then it jumps across the *would-be-singularity* by an amount that depends on $b\ell_p$ when the expressions (41) and (42) are used. The result is illustrated in the Figure 8. To a leading order in Λ (or equivalently in $b\ell_p$) we have

$$\delta S_{CG} = \log(2) - \frac{9\Delta^2 \ell_p^4}{8192\pi^4 \gamma^4 \mu^4} \Lambda^2 + \mathscr{O}(\Lambda^2 \ell_p^4). \tag{C6}$$

The reason for this is the dynamical difference in the evolution of the 1 and 2 components of the wave function. It is important to point out that the entropy jump is generic: the qualitative result would remain the same if we had solved a more realistic model with a scalar field (31) or any other matter coupling.

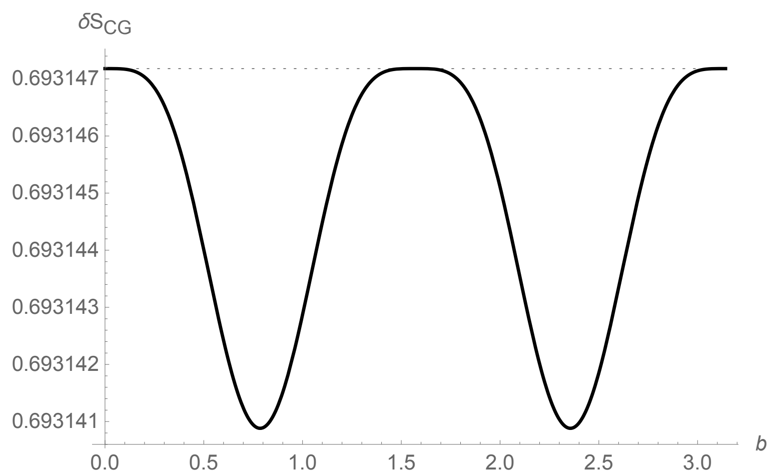


Figure 8. Coarse-graining entropy jumps $\delta S_{\rm CG}$ due to matter coupling (modeled by our simple interaction) after the big bang as a function of $b\ell_p$ for $\gamma = \mu = \Delta_- = 1$. The indistinguishability of the Γ^1 and Γ^2 components of the wave function for a coarse observer introduces an information loss that is quantified by the coarse-graining entropy jump. The coarse-graining entropy is constant $S^{in}_{\rm CG} = \log(2)$ before and evolves to $S^{out}_{\rm CG} = \log(2) + \delta S_{\rm CG}$.

Appendix D: The free massless scalar field (numerical results)

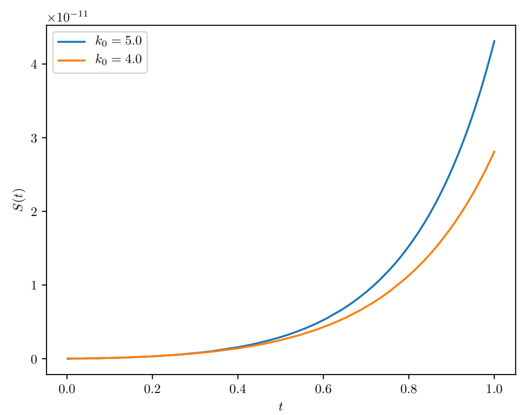
The case of gravity coupled to a scalar field (32) can be solved numerically. The Hamiltonian is given by

$$\hat{H}\psi(\nu) = \frac{3V_0}{32\pi G}\frac{1}{\gamma^2\lambda^2}\left[\Psi(\nu-4\lambda)+\Psi(\nu+4\lambda)-2\Psi(\nu)\right] + \frac{V_0 p_\phi}{(2\pi\ell_{Pl}^2\gamma)^2\nu^2}(1-\delta_{\nu 0})\Psi(\nu), \tag{D1}$$

where the factor $(1-\delta_{\nu 0})$ in potential is there to regularize the big-bang divergence at $\nu = 0$ (see [1, 2]). Qualitative results remain the same for any of the regularization prescriptions discussed in the literature (in particular for those introduced in (79)).

As in the case of the ultra-local Hamiltonian (36), when the initial state is an eigenstate of the field momentum operator p_ϕ, the problem reduces to a scattering problem. The setup of the problem is the same as in Section II D: for simplicity we consider only states supported on the superposition of two lattices $\Gamma_1 = \Gamma_\Delta^{\epsilon=0}$ and $\Gamma_2 = \Gamma_\Delta^{\epsilon=2\lambda}$. Initial states are Gaussian wave packets shifted 2λ one from the other. In other words, if $\phi(\nu,\Gamma_1)$, $\phi(\nu,\Gamma_2)$ are Gaussian wave packets with support on the lattices $\phi(\nu,\Gamma_1)$ and $\phi(\nu,\Gamma_2)$, respectively, we have that $\exp(i\lambda b) \triangleright \phi(\nu,\Gamma_1) = \phi(\nu,\Gamma_2)$. $\phi(\nu,\Gamma_1)$ is peaked around ν_0 while $\phi(\nu,\Gamma_2)$ is peaked around $\nu_0 - 2\lambda$.

We see that entropy grows gradually as the universe approaches the bounce where the matter Hamiltonian starts changing abruptly from one lattice site to another. Thus, components of the quantum state supported on different lattices will 'see' different potentials and entropy will grow rapidly. This effect depends on the semiclassical curvature of the initial state. Initially, when the universe is sufficiently large and for a low cosmological constant Λ, decoherence will become important closer to the bounce than for states with a higher Λ. The wavefunction of states with a higher cosmological constant oscillates faster thus amplifying differences between the matter Hamiltonian when proved by two lattices with different ϵ. This behavior is illustrated in Figure 9.



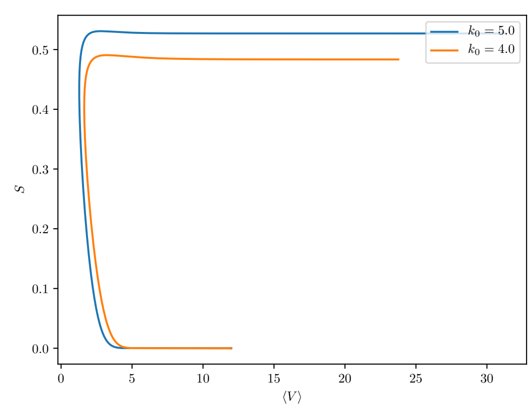


Figure 9. **Left Panel:** Entropy S with respect to unimodular time t away from the bounce, where the matter interaction is almost negligible. For states with a higher cosmological constant, entropy grows faster. **Right Panel:** Entropy S with respect to the expected value of the volume $\langle V\rangle$ through the bounce. The entropy jump around the bounce is generic while the quantitative details depends on the initial state and the value p_ϕ. For both cases $\gamma = 1,\ V_0 = 1,\ \nu_0 = 12,\ p_\phi = 2, and \lambda = \sqrt{2}/40$ in natural units.

[1] Edward Wilson-Ewing. Lattice loop quantum cosmology: scalar perturbations. *Class. Quant. Grav.*, 29:215013, 2012.

[2] Parampreet Singh and Edward Wilson-Ewing. Quantization ambiguities and bounds on geometric scalars in anisotropic loop quantum cosmology. *Class. Quant. Grav.*, 31:035010, 2014.