SUPPLEMENTARY MATERIAL: DETAILED RESULTS FOR MNIST DATASET

We report below the detailed evaluation of the performance for MNIST case-study with missing data patterns with N_S WxW gaps, with $N_s = 20, 30, 6$ and $W = 3, 3, 9$. The results for configuration $N_s = 30$ and $W = 3$ are reported in the main text (Tab.1).

New MNIST	Model	I-score	R-score	AE-score	C-score
$N_S = 20$	DINEOF	-9.41% (-10.95%)	21.54% (20.48%)	64.36% (65.11%)	96.23%
$W = 5$	DINConvAE	55.98% (55.39%)	80.98% (80.58%)	**93.42% (92.35%)**	**98.12%**
	FP(1)-ConvAE	61.79% (61.63%)	82.22% (81.64%)	87.64% (87.56%)	97.55%
	FP(15)-ConvAE	74.99% (72.80%)	88.78% (87.31%)	91.62% (91.13%)	97.96%
	G(14)-ConvAE	**76.50% (75.56%)**	**89.81% (88.81%)**	91.77% (91.21%)	97.91%
$N_S = 30$	DINEOF	-8.86% (-10.19%)	13.89% (12.71%)	64.36% (65.11%)	96.23%
$W = 5$	ConvAE	38.32% (38.16%)	67.42% (67.32%)	**93.42% (92.35%)**	**98.12%**
	Zero-ConvAE	53.69% (53.44%)	74.97% (74.44%)	85.67% (85.83%)	97.03%
	FP(15)-ConvAE	69.27% (67.68%)	83.81% (82.54%)	90.22% (90.04%)	97.59%
	G(14)-ConvAE	**69.82% (68.52%)**	**84.96% (83.76%)**	90.98% (90.66%)	97.45%
$N_S = 6$	DINEOF	-37.47% (-40.00%)	16.83% (15.50%)	64.36% (65.11%)	96.23%
$W = 9$	ConvAE	-27.02% (-28.28%)	46.95% (46.44%)	**93.42% (92.35%)**	**98.12%**
	Zero-ConvAE	-9.94% (-12.03%)	55.41% (54.09%)	86.52% (86.73%)	97.33%
	FP(15)-ConvAE	**26.90% (22.56%)**	**71.18%** (68.45%)	91.03% (90.41%)	97.71%
	G(10)-ConvAE	26.18% **(24.73%)**	70.70% **(69.58%)**	90.30% (90.23%)	97.86%

Table 4. Performance of AE schemes in presence of missing data for MNIST dataset: this table complements the results reported in Tab.1 for other missing data patterns, namely with $N_S = 20, W = 5$, $N_S = 30, W = 5$ and $N_S = 6, W = 9$. We let the reader to the main text and Tab.1 for additional details.

SUPPLEMENTARY MATERIAL: DETAILED RESULTS FOR SST CASE-STUDY

SST	Model	I-Score	R-score	AE-score
	OI	67.59% (57.29%)	70.97% (61.00%)	-
AE models	FP(5)-EOF(20)	32.52% (39.22%)	34.94% (30.39%)	74.17% (56.00%)
	FP(5)-EOF(80)	28.01% (34.83%)	30.91% (25.28%)	**89.95%** (64.53%)
	Zero-ConvAE$_1$	89.12% (86.98%)	89.65% (87.33%)	67.42% (60.41%)
	FP(10)-ConvAE$_1$	87.63% (85.24%)	89.82% (87.28%)	83.81% (77.20%)
	G(8)-ConvAE$_1$	89.08% (87.89%)	89.51% (88.25%)	84.22% (76.32%)
	Zero-ConvAE$_2$	86.70% (86.37%)	87.14% (86.87%)	67.20% (54.77%)
	FP(10)-ConvAE$_2$	88.71% (85.02%)	89.14% (85.49%)	86.24% **(80.76)**
	G(8)-ConvAE$_2$	90.47% (88.00%)	90.98% (88.39%)	86.33% (78.33%)
GENN models	Zero-GENN$_1$	85.46% (79.39%)	86.71% (80.30%)	-94.84% (-172.68%)
	FP(15)-GENN$_1$	89.22% (87.45%)	90.07% (88.50%)	92.61% (90.18%)
	G(12)-GENN$_1$	89.83% **(89.16%)**	90.56% **(90.00%)**	92.23% (90.98%)
	FP(1)-GENN$_2$	86.60% (77.38%)	87.48% (78.01%)	-141.64% (-235.50%)
	FP(15)-GENN$_2$	90.56% (85.93%)	91.33% (87.26%)	**93.04%** **(91.17%)**
	G(12)-GENN$_2$	**91.10%** (87.98%)	**91.83%** (88.81%)	92.36% (90.37%)

Table 5. Performance on SST dataset: We evaluate for each model interpolation, reconstruction and auto-encoding scores, resp. I-score, R-score and AE-score, in terms of percentage of explained variance resp. for the interpolation of missing data areas, the reconstruction of the whole image with missing data and the reconstruction of gap-free images. For each model, we evaluate these score for the training data (first row) and the test dataset (second row in brackets). We consider four different auto-encoder models, namely 20 and 80-dimensional EOFs and ConvAE$_{1,2}$ models, and two GENN models, GENN$_{1,2}$, combined with three interpolation strategies: the classic zero-filling strategy (Zero) and proposed iterative fixed-point (FP) and gradient-based (G) schemes, the figure in brackets denoting the number of iterations. For instance, FP(10)-GENN$_1$ refers to GENN$_1$ with a 10-step fixed-point interpolation scheme. The EOFs are trained from gap-free data. We also consider an Optimal Interpolation (OI) with a space-time Gaussian covariance with empirically-tuned parameters. We refer the reader to the main text for the detailed parameterization of the considered models.