**Design and performance of** **a** **compact air-breathing jet hybrid-electric engine coupled with solid oxide fuel cells**

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## **Supplementary materials S. Basic equations for the Fuel cell model**

 Steam reforming reaction and water gas shift reaction occurs in the SOFC anode channel, which utilizes the water steam from the electrochemical reaction [32]. Energy and mass conservation equations for these reactions are listed in Table S.1.

Table S.1 Chemical reaction equations in the SOFC anode [32, 34]

|  |  |  |
| --- | --- | --- |
| Components | Equations | NO. |
| Water-gas shifting reaction | $$CO+H\_{2}O\leftrightarrow CO\_{2}+H\_{2} Q\_{sh}$$ | (S.1) |
| Methane water reaction  | $$CH\_{4}+H\_{2}O=CO+3H\_{2} Q\_{r}$$ | (S.2) |
| Electrochemical reaction | $H\_{2}+0.5O\_{2}\rightarrow H\_{2}O (T\_{cell}∙∆S+∆G)$  | (S.3) |
| Electrochemical reactions heat | $$Q\_{elec}=T\_{cell}∙∆S-j∙\left(η\_{ohmi}+η\_{conc}+η\_{acti,anode}+η\_{acti,cathode}\right)$$ | (S.4) |
| Mass balance equation in fuel cell  | $$M\_{i,in}+\sum\_{k}^{}C\_{i,k}r\_{k}=M\_{i,out}$$ | (S.5) |
| Energy balance equation in fuel cell  | $$\sum\_{i}^{}m\_{i,in}c\_{p,i}T\_{in}-P\_{cell}=\sum\_{j}^{}m\_{j,out}c\_{p,j}T\_{cell}$$ | (S.6) |

 Electrochemical reactions occur in the three-phase boundaries of fuel cells. The concentrations of working fluids are calculated by porous-media gas-phase transport models [33] as shown in Table S.2.

Table S.2 Concentration equations for reactant and product at the TPB boundary [33].

|  |  |  |
| --- | --- | --- |
| Components | Equations | NO. |
| The partial pressure of H2 at TPB boundary | $$p\_{H\_{2},TPB}=p\_{H\_{2},f}-\frac{RT\_{cell}τ\_{anode}}{2F\overbar{D\_{eff,anode}}}j$$ | (S.7) |
| The partial pressure of H2O at TPB boundary | $$p\_{H\_{2}O,TPB}=p\_{H\_{2}O,f}+\frac{RT\_{cell}τ\_{anode}}{2F\overbar{D\_{eff,anode}}}j$$ | (S.8) |
| The partial pressure of O2 at TPB boundary | $$p\_{O\_{2},TPB}=p-\left(p-p\_{O\_{2},a}\right)exp\left(\frac{RT\_{cell}τ\_{cathode}}{4F\overbar{D\_{eff,cathode}p}}j\right)$$ | (S.9) |

 The open-circuit voltage is the maximum voltage that can be achieved by a fuel cell as (S.10) in Table S.3. Polarization loss includes ohmic, concentration, and activation polarization in Table S.3, which leads to the decline of the open-circuit voltage.

Table S.3 Polarization reaction equations for fuel cells [33, 35-37]

|  |  |  |
| --- | --- | --- |
| Components | Equations | NO. |
| Open-circuit voltage | $$U^{OCP}=U\_{H\_{2}}^{0}-\frac{RT\_{cell}}{2F}ln\left(\frac{p\_{H\_{2}O,TPB}}{p\_{H\_{2},TPB}·p\_{O\_{2},TPB}^{{1}/{2}}}\right)$$ | (S.10) |
| Actual voltage | $U=U^{OCP}-$($η\_{ohmi}+η\_{conc}+η\_{act,anode}+η\_{act,cathode}$) | (S.11) |
| Ohmic polarization | $$η\_{ohmi}=jR\_{ohmi}=j\left(\frac{τ\_{anode}}{σ\_{anode}}+\frac{τ\_{electrolyte}}{σ\_{electrolyte}}+\frac{τ\_{cathode}}{σ\_{cathode}}\right)$$ | (S.12) |
| Concentration polarization | $$η\_{conc}=\frac{RT\_{cell}}{2F}ln\left(\frac{p\_{H\_{2}O,TPB}p\_{H\_{2},f}}{p\_{H\_{2},TPB}p\_{H\_{2}O,f}}\right)+\frac{RT\_{cell}}{4F}ln\left(\frac{p\_{O\_{2},a}}{p\_{O\_{2},TPB}}\right)$$ | (S.13) |
| Anode activation polarization | $$η\_{acti,anode}=\frac{2RT\_{cell}}{Fn\_{e}}sinh^{-1}\left({j}/{\left(2j\_{0,anode}\right)}\right)$$ | (S.14) |
| Cathode activation polarization | $$η\_{acti,cathode}=\frac{2RT\_{cell}}{Fn\_{e}}sinh^{-1}\left({j}/{\left(2j\_{0,cathode}\right)}\right)$$ | (S.15) |
| Anode exchange density | $$j\_{0,anode}=\frac{RT\_{cell}}{nF}k\_{anode}exp\left(-\frac{E\_{anode}}{RT\_{cell}}\right)$$ | (S.16) |
| Cathode exchange density | $$j\_{0,cathode}=\frac{RT\_{cell}}{nF}k\_{cathode}exp\left(-\frac{E\_{cathode}}{RT\_{cell}}\right)$$ | (S.17) |

 The performance parameters of SOFCs include power, efficiency, and fuel utilization, which are defined in Table S.4.

Table S.4 Performance equations for fuel cells

|  |  |  |
| --- | --- | --- |
| Components | Equations | NO. |
| Output power | $$P\_{cell}=U·j·L·W$$ | (S.18) |
| Electric efﬁciency | $$η\_{cell}=\frac{P\_{cell}}{\left(y\_{CH\_{4}}LHV\_{CH\_{4}}+y\_{H\_{2}}LHV\_{H\_{2}}+y\_{CO}LHV\_{CO}\right)M\_{fuel}}=η\_{f}·\frac{ΔG}{ΔH}·\frac{U}{U^{OCP}}$$ | (S.19) |
| Fuel utilization | $$ϕ={(j×L×W)}/{(8Fn\_{CH\_{4}}+2Fn\_{H\_{2}}+2Fn\_{CO})}$$ | (S.20) |

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