**Appendix A**

# The problem-solving training program

The training program focused on fostering student reflection on external representations that they had individually developed and used to solve challenging word problems. Specifically, during the work phase, students were encouraged and motivated to solve word problems by working on them with one (or more) external representation(s). Afterwards, selected student representations were reflected on and discussed in class. The teacher, in this case the first author, accompanied the students during the development and reflection phase.

External representations are assigned a significant role, not only from a didactic but also from a psychological point of view. The act of externalizing can support individual thought and solution processes, whether on a depictional or descriptive level (Amarel, 1966; Reisberg, 1987). Students who represent their thought processes and solution ideas externally relieve their working memory and gain capacities which they can use for problem solving (Cox, 1999; Kirsh, 2010; Sweller, 1994).

The advantage of external representations is that they are more stable than mental representations (Schnotz et al., 2010). As they can be constantly changed, supplemented by new information, revised and adapted, they offer the individual a high degree of flexibility and agility in restructuring and limiting the problem area (Newell & Simon, 1972; Reisberg, 1987). Furthermore, external representations support and facilitate the structuring of thought processes and the individual acquisition of knowledge (Reisberg, 1987). They lead the learner to the solution if he or she succeeds in representing solution-relevant features and selecting irrelevant ones (Norman, 1993) and, on this basis, in activating the required prior knowledge. In the process of finding adequate representations, the students can benefit from the approaches of their classmates. The students’ representations reveal their individual understanding, can be referred to appreciatively in class and further developed. By examining different solutions and representations, invisible mathematical knowledge and identified structures become accessible to all learners in a way that is easy to understand (Duval, 1999).

The apparent differences that exist can be used above all to explicitly elaborate and discuss the aspects of the respective representations that are conducive to learning, and also to make comparisons and identify commonalities. On this basis, advantages and disadvantages of individual steps can be presented and compared, and individual approaches can be assessed and evaluated regarding their suitability (Lester, 1985). The learners experience when and why which forms of representation are suitable for solving a wide variety of problems and gradually build up an individual repertoire of representations which enables them to select suitable representations for specific tasks (Cox, 1999; Cox & Brna, 1995). Thus, with the help of representations and growing experience, the solvers can succeed in identifying the mathematical structures of the tasks, modeling them, mathematizing them and, last but not least, solving the task successfully (Sturm, 2018, 2019).