



Supplementary Figure 2. Phase transition of orientation representation.

Orientation representation undergoes a sort of phase transition at  $\beta = \beta_c$  from the random representation phase ( $\beta < \beta_c$ ) to the orderly orientation map phase ( $\beta > \beta_c$ ), when the value of the inverse temperature  $\beta$  increases from 0. The top row shows the orientation representations at different values of  $\beta$ . The middle row indicates the radial power spectra of respective orientation representations. The bottom figure illustrates the  $\beta$ -dependence of the maximum power. The maximum power changes almost linearly within the interval of  $30 \leq \beta \leq 45$ , as indicated by the red line. The transition point  $\beta_c$  is reasonably determined by the intersection of the red line with the horizontal axis, and we obtain  $\beta_c \approx 28$  in the parameter setting of simulations that we performed in this study.