APPENDIX

A GENERAL EXPRESSIONS FOR GAUGE AND YUKAWA BETA-FUNCTIONS

For completeness, we collect the literature on expressions for $\overline{\text{MS}}$-scheme 3-loop gauge and 2-loop Yukawa beta-functions of general non-Abelian gauge-Yukawa theories. One can straightforwardly reduce *one* non-Abelian subgroup to an Abelian subgroup. Note that this does *not* directly generalize to multiple Abelian factors because of kinetic mixing, cf. [113, 114]. We neglect all contributions from scalar quartic couplings. The 1-loop beta-functions for a simple non-Abelian gauge group were first calculated along in [3, 4] and subsequently generalized to 2-loop and semi-simple groups in [74, 75] and [76, 77, 79], respectively. State-of-the-art 3-loop results have been obtained in [82, 84] for simple and in [91] for semi-simple groups. For the Yukawa couplings, 1-loop results were first derived in [5] and 2-loop results in [78, 80, 81]. Specific results for the Standard Model were derived in [83, 85, 86] at 2-loop, in [87–90] at 3-loop order, and in [115] partially at 4-loop order. Neglecting contributions from other couplings the simple gauge beta-functions have been calculated up to 5-loop order [116–118]. Finally, SARAH [119, 120] and PyR@TE [121, 122] provide computer algebra tools for general perturbatively renormalizable beta-functions at two-loop level, see also [92], as well as [93] for general results at three-loop.

For the present purpose, it is sufficient to work with the 3-loop gauge beta functions (including general Yukawa-coupling contributions but neglecting quartic couplings) and with the 2-loop Yukawa beta functions (again neglecting quartic couplings). Focusing on a simple gauge group with and one fermionic representation R_F, the Lagrangian reads

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F_A^{\mu\nu}F^A_{\mu\nu} + \mathcal{L}_{(R_F)} \quad \text{with} \\ \mathcal{L}_{(R_F)} &= +\frac{1}{2}D^\mu\phi_a D_\mu\phi_a + i\psi_j^\dagger\sigma^\mu D_\mu\psi_j - \frac{1}{2}\left(Y^a_{jk}\psi_j\zeta\psi_k\phi_a + Y^{a*}_{jk}\psi_j^\dagger\zeta\psi_k^\dagger\phi_a\right) .\end{aligned} \tag{18}$$

Here, we have already assumed that each fermionic representation R_F is accompanied by a suitable set of scalars to facilitate Yukawa couplings Y^a_{ij}. Here, $F^A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g\, f^{ABC} A^b_\mu A^c_\nu$ is the standard field-strength tensor with gauge group structure constants f^{ABC} and $\zeta = i\sigma_2$ with σ_2 the 2nd Pauli matrix. Fermions and scalars are minimally coupled via covariant derivatives corresponding with generators t^A_{ij} and θ^A_{ab}, respectively, i.e.,

$$D_\mu\phi_a = \partial_\mu\phi_a - i\, g\, \theta^A_{ab}\, A^A_\mu\, \phi_b\ , \tag{19}$$

$$D_\mu\psi_i = \partial_\mu\psi_i - i\, g\, t^A_{ij}\, A^A_\mu\, \psi_j\ . \tag{20}$$

Finally, Y^a_{ij} denote complex Yukawa coupling matrices. A generalization to multiple gauge groups and representations is straightforward. We have omitted quartic couplings and mass terms since we neglect them for this paper.

Most of the contributions to the beta-functions can be written in terms of the respective quadratic Casimirs C_2 and Dynkin indices S_2. For the adjoint gauge fields, fermions, and scalars they read

$$C_2^{\text{adj}}\delta^{AB} = f^{ACD}f^{BCD}\ , \qquad C_2^{R_F} = \sum_{A=1}^{d} t^A t^A\ , \qquad C_2^{R_S} = \sum_{A=1}^{d}\theta^A\theta^A\ , \tag{21}$$

$$S_2^{R_F}\delta^{AB} = \text{Tr}\Big[t^A t^B\Big]\ , \qquad S_2^{R_S}\delta^{AB} = \text{Tr}\Big[\theta^A\theta^B\Big]\ . \tag{22}$$

We denote the associated beta-functions by

$$\beta_g = \left[\frac{\beta_g^{\text{(1-loop)}}}{(4\pi)^2} + \frac{\beta_g^{\text{(2-loop)}}}{(4\pi)^4} + \frac{\beta_g^{\text{(3-loop)}}}{(4\pi)^6}\right] , \tag{23}$$

$$\beta_{Y^a} = \left[\frac{\beta_a^{\text{(1-loop)}}}{(4\pi)^2} + \frac{\beta_a^{\text{(2-loop)}}}{(4\pi)^4}\right] . \tag{24}$$

The explicit expressions can be found in [84] and [92], respectively. Note, that we omit contributions from scalar quartic couplings. The replacement rules to generalize to semi-simple gauge groups at 2-loop level can be found in [85]. All the required beta-functions can thereby be derived from this general set of expressions.

B REDUCTION TO (B)SM BETA-FUNCTIONS

To obtain the beta-functions for the BSM extensions covered in Section 6, we specialize to the SM Lagrangian supplemented by $\mathcal{N}$ different types of vector-like BSM fermions $\Psi^{(I)}$ in the representation R_I, where $I = 1, \ldots, \mathcal{N}$ labels the different types of BSM representations, each of which can come with a multiplicity N_{R_I}. For each such type of vectorlike BSM fermions, we also include an $N_{R_I} \times N_{R_I}$ matrix of uncharged complex scalars $\phi_{(I)}$ which allows for Yukawa couplings. The corresponding Lagrangian reads

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{I=1}^{\mathcal{N}} \left[\text{Tr}\left(\overline{\psi}^{(I)} i\not{D}\psi^{(I)}\right) + \text{Tr}\left(\partial_\mu \phi^\dagger_{(I)} \partial^\mu \phi_{(I)}\right) + y_I \text{Tr}\left(\overline{\psi}_L^{(I)} \phi_{(I)} \psi_R^{(I)} + \overline{\psi}_R^{(I)} \phi^\dagger_{(I)} \psi_L^{(I)}\right)\right] , \tag{25}$$

where we have decomposed the BSM fermions into two-component left- and right-handed parts, i.e., $\psi^{(I)}_{R/L} = \frac{1}{2}(1 \pm \gamma_5)\psi^{(I)}$. We neglect any quartic couplings. Therefore, the different types of BSM fermions couple to each other as well as to the SM only via their minimal gauge interactions. The matrix-Yukawa couplings in Eq. (18) are diagonal and therefor the different traces of Yukawa-matrices in the beta-functions, cf. [84, 92], simplify. Introducing, for the gauge couplings $\alpha_i = \frac{g_i^2}{(4\pi)^2}$ for $i = 1, 2, 3$, for the top-Yukawa coupling $\alpha_t = \frac{y_t^2}{(4\pi)^2}$, and for the BSM Yukawa couplings $\alpha_I = \frac{y_{(I)}^2}{(4\pi)^2}$, the general NLO and NNLO contributions to the beta-functions reduce to the follwing form

$$\beta^{\text{(NLO)}}_{\alpha_i} = \alpha_i^2 \left[B_i + \sum_{j=1}^{3} C_{ij}\alpha_j - \sum_{I=0}^{\mathcal{N}} D_{iI}\alpha_I\right] , \tag{26}$$

$$\beta^{\text{(NLO)}}_{\alpha_I} = \alpha_I \left[E_I \alpha_I - \sum_{j=1}^{3} F_{Ij}\alpha_j\right] , \tag{27}$$

$$\beta^{\text{(NNLO)}}_{\alpha_i} = \alpha_i^2 \left[\sum_{j=1}^{3}\sum_{k=j}^{3} M_{ijk}\alpha_j\alpha_k - \sum_{j=1}^{3}\sum_{I=0}^{\mathcal{N}} K_{ijI}\alpha_j\alpha_I + \sum_{I=0}^{\mathcal{N}} \overline{K}_{iI}\alpha_I^2\right] , \tag{28}$$

$$\beta^{\text{(NNLO)}}_{\alpha_I} = \alpha_I \left[\sum_{j=1}^{3} V_{Ij}\alpha_j\alpha_I + \overline{V}_I \alpha_I^2 + \sum_{i=1}^{3}\sum_{j=i}^{3} W_{Ijk}\alpha_j\alpha_k\right] , \tag{29}$$

where the notation includes the SM top-Yukawa coupling as $\alpha_t = \alpha_{I=0}$. The group theoretic coefficients B_i, C_{ij}, D_{iI}, E_I, F_{Ij} at NLO level and M_{ijk}, K_{ijI}, $\overline{K}_{iI}$, V_{Ij}, $\overline{V}_I$, W_{Ijk} at NNLO level are well-known for the SM [83, 85–90]. The BSM contributions can straightforwardly be generalized from the expressions for a single type of BSM representation, cf. [40], by suitable summation. For completeness, we list them below, where the Casimirs $C_{N_C}^{R_I}$, Dynkin indices $S_{N_C}^{R_I}$, and dimensions $d_{N_C}^{R_I}$ for the BSM representations R_I of the non-Abelian SM gauge groups $SU(N_C)$ with $N_C = 2$ and $N_C = 3$ are given by

$$C_2^{R_I} = \ell(\ell+1)\,, \qquad d_2^{R_I} = 2\ell + 1\,, \qquad S_2^{R_I} = \frac{C_2^{R_I} d_2^{R_I}}{3}\,, \tag{30}$$

$$C_3^{R_I} = \frac{1}{2}(p+1)(q+1)(p+q+2)\,, \qquad d_3^{R_I} = p + q + \frac{1}{3}\left(p^2 + q^2 + p\,q\right)\,, \qquad S_3^{R_I} = \frac{C_3^{R_I} d_3^{R_I}}{8}\,, \tag{31}$$

where $l = 1/2,\, 1,\, 3/2,\, \ldots$ and $p, q = 1,\, 2,\, 3,\, \ldots$ denote the highest weights for $SU(2)$ and $SU(3)$, respectively.

The group-theoretic NLO, i.e., 1-loop and 2-loop, coefficients in the gauge beta functions of the SM including BSM representations read

$$B_1 = \frac{41}{3} + \frac{8}{3}\sum_{I=1}^{\mathcal{N}} N_{R_I} Y_{R_I}^2 d_2^{R_I} d_3^{R_I} \qquad B_2 = -\frac{19}{3} + \frac{8}{3}\sum_{I=1}^{\mathcal{N}} N_{R_I} S_2^{R_I} d_3^{R_I}$$

$$B_3 = -14 + \frac{8}{3}\sum_{I=1}^{\mathcal{N}} N_{R_I} S_3^{R_I} d_2^{R_I} \qquad C_{11} = \frac{199}{9} + 8\sum_{I=1}^{\mathcal{N}} Y_{R_I}^4 N_{R_I} d_2^{R_I} d_3^{R_I}$$

$$C_{12} = 9 + 8\sum_{I=1}^{\mathcal{N}} Y_{R_I}^2 N_{R_I} C_2^{R_I} d_2^{R_I} d_3^{R_I} \qquad C_{13} = \frac{88}{3} + 8\sum_{I=1}^{\mathcal{N}} N_{R_I} Y_{R_I}^2 C_3^{R_I} d_2^{R_I} d_3^{R_I}$$

$$C_{21} = 3 + 8\sum_{I=1}^{\mathcal{N}} N_{R_I} Y_{R_I}^2 S_2^{R_I} d_3^{R_I} \qquad C_{22} = \frac{35}{3} + 4\sum_{I=1}^{\mathcal{N}} N_{R_I} S_2^{R_I} d_3^{R_I}\left(2\,C_2^{R_I} + \frac{20}{3}\right)$$

$$C_{23} = 24 + 8\sum_{I=1}^{\mathcal{N}} N_{R_I} S_2^{R_I} C_3^{R_I} d_3^{R_I} \qquad C_{31} = \frac{11}{3} + 8\sum_{I=1}^{\mathcal{N}} N_{R_I} Y_{R_I}^2 S_3^{R_I} d_2^{R_I}$$

$$C_{32} = 9 + 8\sum_{I=1}^{\mathcal{N}} N_{R_I} S_3^{R_I} C_2^{R_I} d_2^{R_I} \qquad C_{33} = -52 + 4\sum_{I=1}^{\mathcal{N}} N_{R_I} S_3^{R_I} d_2^{R_I}(2\,C_3^{R_I} + 10)$$

$$D_{1I} = 4\sum_{I=1}^{\mathcal{N}} N_{R_I}^2 Y_{R_I}^2 d_2^{R_I} d_3^{R_I} \qquad D_{2I} = \frac{1}{3}4\sum_{I=1}^{\mathcal{N}} N_{R_I}^2 C_2^{R_I} d_2^{R_I} d_3^{R_I}$$

$$D_{3I} = \frac{1}{8}4\sum_{I=1}^{\mathcal{N}} N_{R_I}^2 C_3^{R_I} d_2^{R_I} d_3^{R_I}$$

The group-theoretic NLO, i.e., 1-loop, coefficients in the Yukawa beta functions of the SM including BSM representations read

$$E_t = 9 \qquad E_I = 2(N_{R_I} + d_2^{R_I} d_3^{R_I})$$

$$F_{t1} = \frac{17}{6} \qquad F_{t2} = \frac{9}{2} \qquad F_{t3} = 16$$
$$F_{I1} = 12Y_{R_I}^2 \qquad F_{I2} = 12C_2^{R_I} \qquad F_{I3} = 12C_3^{R_I}$$

The group-theoretic NNLO, i.e., 3-loop, coefficients in the gauge beta functions of the SM including BSM representations read

$$M_{111} = \frac{388613}{2592} + \sum_{I=1}^{\mathcal{N}} \left[\frac{4405}{162} N_{R_I} Y_{R_I}^2 d_2^{R_I} d_3^{R_I} + \frac{463}{9} N_{R_I} Y_{R_I}^4 d_2^{R_I} d_3^{R_I} + 4 N_{R_I} Y_{R_I}^6 d_2^{R_I} d_3^{R_I} + \frac{88}{9} N_{R_I}^2 Y_{R_I}^6 \left(d_2^{R_I} d_3^{R_I} \right)^2 \right]$$

$$M_{122} = \frac{1315}{32} + \sum_{I=1}^{\mathcal{N}} \left[\frac{245}{9} C_2^{R_I} N_{R_I} Y_{R_I}^2 d_2^{R_I} d_3^{R_I} - 4 \left(C_2^{R_I} \right)^2 N_{R_I} Y_{R_I}^2 d_2^{R_I} d_3^{R_I} + \frac{23}{2} N_{R_I} S_2^{R_I} d_3^{R_I} \right]$$

$$M_{133} = 198 + \sum_{I=1}^{\mathcal{N}} \left[\frac{178}{3} C_3^{R_I} N_{R_I} Y_{R_I}^2 d_2^{R_I} d_3^{R_I} - 4 \left(C_3^{R_I} \right)^2 N_{R_I} Y_{R_I}^2 d_2^{R_I} d_3^{R_I} - \frac{968}{27} N_{R_I} S_3^{R_I} d_2^{R_I} \right]$$

$$M_{112} = \frac{205}{48} - \sum_{I=1}^{\mathcal{N}} 8\, C_2^{R_I} N_{R_I} Y_{R_I}^4 d_2^{R_I} d_3^{R_I}$$

$$M_{113} = \frac{274}{27} + \sum_{I=1}^{\mathcal{N}} 8\, C_3^{R_I} N_{R_I} Y_{R_I}^4 d_2^{R_I} d_3^{R_I}$$

$$M_{123} = 2 + \sum_{I=1}^{\mathcal{N}} 8\, C_2^{R_I} C_3^{R_I} N_{R_I} Y_{R_I}^2 d_2^{R_I} d_3^{R_I}$$

$$M_{211} = \frac{5597}{288} + \sum_{I=1}^{\mathcal{N}} \left[\frac{23}{6} N_{R_I} Y_{R_I}^2 d_2^{R_I} d_3^{R_I} + \frac{463}{9} Y_{R_I}^2 N_{R_I} S_2^{R_I} d_3^{R_I} + 4 N_{R_I} Y_{R_I}^4 S_2^{R_I} d_3^{R_I} \right]$$

$$M_{222} = \frac{324953}{864} + \sum_{I=1}^{\mathcal{N}} \left[\frac{13411}{54} N_{R_I} S_2^{R_I} d_3^{R_I} + \frac{533}{9} N_{R_I} C_2^{R_I} S_2^{R_I} d_3^{R_I} - 4 N_{R_I} \left(C_2^{R_I} \right)^2 S_2^{R_I} d_3^{R_I} \right]$$

$$M_{233} = 162 + \sum_{I=1}^{\mathcal{N}} \left[\frac{178}{3} C_3^{R_I} N_{R_I} S_2^{R_I} d_3^{R_I} - 4 \left(C_3^{R_I} \right)^2 N_{R_I} S_2^{R_I} d_3^{R_I} - \frac{88}{3} N_{R_I} S_3^{R_I} d_2^{R_I} \right]$$

$$M_{212} = \frac{291}{16} + \sum_{I=1}^{\mathcal{N}} \left[32\, Y_{R_I}^2 N_{R_I} S_2^{R_I} d_3^{R_I} - 8\, Y_{R_I}^2 C_2^{R_I} N_{R_I} S_2^{R_I} d_3^{R_I} \right]$$

$$M_{213} = \frac{2}{3} + \sum_{I=1}^{\mathcal{N}} 8\, Y_{R_I}^2 C_3^{R_I} N_{R_I} S_2^{R_I} d_3^{R_I}$$

$$M_{223} = 78 + \sum_{I=1}^{\mathcal{N}} \left[32\, C_3^{R_I} N_{R_I} S_2^{R_I} d_3^{R_I} - 8\, C_2^{R_I} C_3^{R_I} N_{R_I} S_2^{R_I} d_3^{R_I} \right]$$

$$M_{211} = \frac{5597}{288} + \sum_{I=1}^{\mathcal{N}} \left[\frac{23}{6} N_{R_I} Y_{R_I}^2 d_2^{R_I} d_3^{R_I} + \frac{463}{9} Y_{R_I}^2 N_{R_I} S_2^{R_I} d_3^{R_I} + 4 N_{R_I} Y_{R_I}^4 S_2^{R_I} d_3^{R_I}\right]$$

$$M_{311} = \frac{2615}{108} + \sum_{I=1}^{\mathcal{N}} \left[\frac{121}{27} N_{R_I} Y_{R_I}^2 d_2^{R_I} d_3^{R_I} + \frac{463}{9} Y_{R_I}^2 N_{R_I} S_3^{R_I} d_2^{R_I} + 4 N_{R_I} Y_{R_I}^4 S_3^{R_I} d_2^{R_I}\right]$$

$$M_{322} = \frac{109}{4} + \sum_{I=1}^{\mathcal{N}} \left[- 11 N_{R_I} S_2^{R_I} d_3^{R_I} + \frac{245}{9} C_2^{R_I} N_{R_I} S_3^{R_I} d_2^{R_I} - 4 \left(C_2^{R_I}\right)^2 N_{R_I} S_3^{R_I} d_2^{R_I}\right]$$

$$M_{333} = 65 + \sum_{I=1}^{\mathcal{N}} \left[\frac{6242}{9} N_{R_I} S_3^{R_I} d_2^{R_I} + \frac{322}{3} N_{R_I} C_3^{R_I} S_3^{R_I} d_2^{R_I} - 4 N_{R_I} \left(C_3^{R_I}\right)^2 S_3^{R_I} d_2^{R_I}\right]$$

$$M_{312} = \frac{1}{4} + \sum_{I=1}^{\mathcal{N}} 8\, Y_{R_I}^2 C_2^{R_I} N_{R_I} S_3^{R_I} d_2^{R_I}$$

$$M_{313} = \frac{154}{9} + \sum_{I=1}^{\mathcal{N}} \left[48\, Y_{R_I}^2 N_{R_I} S_3^{R_I} d_2^{R_I} - 8\, Y_{R_I}^2 C_3^{R_I} N_{R_I} S_3^{R_I} d_2^{R_I}\right]$$

$$M_{323} = 42 + \sum_{I=1}^{\mathcal{N}} \left[48\, C_2^{R_I} N_{R_I} S_3^{R_I} d_2^{R_I} - 8\, C_2^{R_I} C_3^{R_I} N_{R_I} S_3^{R_I} d_2^{R_I}\right]$$

as well as

$$K_{11I} = 6\, Y_{R_I}^4 N_{R_I}^2 d_2^{R_I} d_3^{R_I} \qquad K_{12I} = 6\, Y_{R_I}^2 C_2^{R_I} N_{R_I}^2 d_2^{R_I} d_3^{R_I}$$

$$K_{13I} = 6\, Y_{R_I}^2 C_3^{R_I} N_{R_I}^2 d_2^{R_I} d_3^{R_I} \qquad K_{21I} = 2\, Y_{R_I}^2 C_2^{R_I} N_{R_I}^2 d_2^{R_I} d_3^{R_I}$$

$$K_{22I} = 16\, C_2^{R_I} N_{R_I}^2 d_2^{R_I} d_3^{R_I} + 2 \left(C_2^{R_I}\right)^2 N_{R_I}^2 d_2^{R_I} d_3^{R_I} \qquad K_{23I} = 2\, C_2^{R_I} C_3^{R_I} N_{R_I}^2 d_2^{R_I} d_3^{R_I}$$

$$K_{31I} = \frac{3}{4} Y_{R_I}^2 C_3^{R_I} N_{R_I}^2 d_2^{R_I} d_3^{R_I} \qquad K_{32I} = \frac{3}{4} C_2^{R_I} C_3^{R_I} N_{R_I}^2 d_2^{R_I} d_3^{R_I}$$

$$K_{33I} = 9\, C_3^{R_I} N_{R_I}^2 d_2^{R_I} d_3^{R_I} + \frac{3}{4} \left(C_3^{R_I}\right)^2 N_{R_I}^2 d_2^{R_I} d_3^{R_I} \qquad \overline{K}_{1I} = 6\, Y_{R_I}^2 N_{R_I}^3 d_2^{R_I} d_3^{R_I} + 7 Y_{R_I}^2 N_{R_I}^2 \left(d_2^{R_I} d_3^{R_I}\right)^2$$

$$\overline{K}_{2I} = 2\, C_2^{R_I} N_{R_I}^3 d_2^{R_I} d_3^{R_I} + \frac{7}{3} C_2^{R_I} N_{R_I}^2 \left(d_2^{R_I} d_3^{R_I}\right)^2 \qquad \overline{K}_{3I} = \frac{3}{4} C_3^{R_I} N_{R_I}^3 d_2^{R_I} d_3^{R_I} + \frac{7}{8} C_3^{R_I} N_{R_I}^2 \left(d_2^{R_I} d_3^{R_I}\right)^2$$

$$K_{11t} = \frac{2827}{144} \qquad K_{12t} = \frac{785}{16} \qquad K_{13t} = \frac{58}{3} \qquad K_{21t} = \frac{593}{48}$$

$$K_{22t} = \frac{729}{16} \qquad K_{23t} = 14 \qquad K_{31t} = \frac{101}{12} \qquad K_{32t} = \frac{93}{4}$$

$$K_{33t} = 80 \qquad \overline{K}_{1t} = \frac{315}{8} \qquad \overline{K}_{2t} = \frac{147}{8} \qquad \overline{K}_{3t} = 30$$

The group-theoretic NNLO, i.e., 2-loop, coefficients in the Yukawa beta functions of the SM including BSM representations read

$$V_{I1} = 2\,(8\, N_{R_I} + 5\, d_2^{R_I} d_3^{R_I}) Y_{R_I}^2 \qquad V_{I2} = 2\,(8\, N_{R_I} + 5\, d_2^{R_I} d_3^{R_I}) C_2^{R_I}$$

$$V_{I3} = 2\,(8\,N_{R_I} + 5\,d_2^{R_I} d_3^{R_I}) C_3^{R_I} \qquad \overline{V}_I = 4 - \frac{1}{2} N_{R_I}^2 + 3\,N_{R_I} d_2^{R_I} d_3^{R_I}$$

$$W_{I11} = \left(\frac{211}{3} - 6 Y_{R_I}^2 + \frac{40}{3} Y_{R_I}^2 N_{R_I} d_2^{R_I} d_3^{R_I} \right) Y_{R_I}^2 \qquad W_{I22} = \left(-\frac{257}{3} - 6 C_2^{R_I} + \frac{40}{3} N_{R_I} S_2^{R_I} d_3^{R_I} \right) C_2^{R_I}$$

$$W_{I33} = \left(-154 - 6 C_3^{R_I} + \frac{40}{3} N_{R_I} S_3^{R_I} d_2^{R_I} \right) C_3^{R_I} \qquad W_{I12} = -12\,Y_{R_I}^2 C_2^{R_I}$$

$$W_{I13} = -12\,Y_{R_I}^2 C_3^{R_I} \qquad W_{I23} = -12\,C_2^{R_I} C_3^{R_I}$$

$$V_{t1} = \frac{131}{8} \qquad V_{t2} = \frac{225}{8}$$

$$V_{t3} = 72 \qquad \overline{V}_t = -24$$

$$W_{t11} = \frac{1187}{108} + \frac{58}{27} Y_{R_I}^2 N_{R_I} d_2^{R_I} d_3^{R_I} \qquad W_{t22} = -\frac{23}{2} + 2 S_2^{R_I} N_{R_I} d_3^{R_I}$$

$$W_{t33} = -216 + \frac{160}{9} S_3^{R_I} N_{R_I} d_2^{R_I} \qquad W_{t12} = \frac{3}{2}$$

$$W_{t13} = \frac{38}{9} \qquad W_{t23} = 18$$

C COEFFICIENTS FOR SIMPLE GAUGE GROUPS

General expressions for the beta functions of the simple gauge groups of the SM with a single type of BSM matter, i.e., $\mathcal{N} = 1$ in Eq. 25, can be extracted from the SM expressions in App. B. We denote the beta function coefficients by

$$\beta_{\alpha_i}^{(\mathrm{NLO})} = \alpha_g^2 \Big[B + C \alpha_g - D \alpha_y \Big] \,, \qquad \beta_{\alpha_i}^{(\mathrm{NNLO})} = \alpha_g^2 \Big[M\,\alpha_g^2 - K\,\alpha_g \alpha_y + \overline{K}\,\alpha_y^2 \Big] \,, \tag{32}$$

$$\beta_{\alpha_y}^{(\mathrm{NLO})} = \alpha_y \Big[E\,\alpha_y - F\,\alpha_y \Big] \,, \qquad \beta_{\alpha_y}^{(\mathrm{NNLO})} = \alpha_y \Big[V\,\alpha_g \alpha_y + \overline{V}\,\alpha_y^2 + W\,\alpha_g^2 \Big] \,, \tag{33}$$

The coefficients can be obtained by subtracting the SM part, e.g., those of U(1) by

$$B \equiv B_1 - B_1|_{N_{R_I}=0} = \frac{8}{3} N_F Y^2 \,, \tag{34}$$

$$C \equiv C_{11} - C_{11}|_{N_{R_I}=0} = 8\,Y^4 N_F \,, \tag{35}$$

$$D \equiv D_{1I} - D_{1I}|_{N_{R_I}=0} = 4 N_F^2 Y^2 \,, \tag{36}$$

$$E \equiv E_I - E_I|_{N_{R_I}=0} = 2(N_F + 1) \,, \tag{37}$$

$$F \equiv F_{I1} - F_{I1}|_{N_{R_I}=0} = 12 Y^2 \,, \tag{38}$$

$$M \equiv M_{111} - B_{111}|_{N_{R_I}=0} = \frac{4405}{162} N_F Y^2 + \frac{463}{9} N_F Y^4 + 4 N_F Y^6 + \frac{88}{9} N_F^2 Y^6 \,, \tag{39}$$

$$K \equiv K_{11I} - K_{11I}|_{N_{R_I}=0} = 6\,Y^4 N_F^2 \,, \tag{40}$$

$$\overline{K} \equiv \overline{K}_{1I} - \overline{K}_{1I}|_{N_{R_I}=0} = 6\,Y^2 N_F^3 + 7 Y^2 N_F^2 \,, \tag{41}$$

where we have reduced to singlets under the other gauge groups, i.e., $d_2^{R_I} = 1 = d_3^{R_I}$, and replaced the notation $N_{R_I} \to N_F$ as well as $Y_{R_I} \to Y$.

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