Supplementary Material: Pore Network Modeling of Core Forming Melts in Planetesimals

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1 MODEL OF MELT SEGGREGATION

During liquid phase sintering of partially molten aggregates, the distribution of melt within a unit volume is controlled by the conservation of mass, momentum, energy, and entropy (Takei and Hier-Majumder, 2009; Sramek et al., 2006). The governing partial differential equations are nonlinear in nature. A number of previous works have studied the numerical solutions to these nonlinear equations and analytical solutions to linearized versions of the governing equations (Bercovici et al., 2001; Ricard et al., 2001; Hier-Majumder et al., 2006; Takei and Hier-Majumder, 2009). We refer the interested reader to these studies for a detailed analysis of the derivation and solution of the governing equations. In this work, we focus on analytical solutions to the linearized equations to explain the experimental observations, similar to the work of King et al. (2011).

The analytical solutions to the governing equations are built using marginal stability analysis. In this analysis, we express the melt fraction in the subsample as a sum of a base state, ϕ_0 and a perturbed state with a wave number k and a complex growth rate λ, expressed as,

$$\phi(x,t) = \phi_0 + \epsilon\tilde{\phi}\mathrm{e}^{-ikx+\lambda t}, \tag{1}$$

where $\tilde{\phi}$ is a constant amplitude of perturbation, x is length, and t is time, and $\epsilon \ll 1$ is a constant coefficient of perturbation. The growth or decay of small perturbations is controlled by λ.

Hier-Majumder et al. (2006) demonstrated that the wetting behavior of the solid-melt, characterized by the dihedral angle at the melt located in grain triple junctions, controls the nature of growth of the perturbations. In wetting melt-solid aggregates, such as olivine and basalt samples (dihedral angle around 30^o, Takei and Hier-Majumder, 2009), surface tension drives melt away from high to low concentrations, a mechanism named as homogenization (Hier-Majumder et al., 2006). King et al. (2011) and Parsons et al. (2008) demonstrated this phenomenon from experimentally annealed synthetic aggregates of olivine and basalt, where the rock was first deformed to created melt-rich bands and then annealed to observe melt moving away from bands. In non-wetting sulfide-olivine melts, the opposite behavior is expected, as the strong surface tension of sulfide melts induce a flow of the melt into pockets of high melt concentration. This behavior is sometimes called self-segregation Hier-Majumder et al. (2006).

In the absence of deformation, the growth or decay of perturbations in melt fraction is controlled by the mechanism of dissolution-precipitation. The growth rate during dissolution precipitation is given by (equation 38, Takei and Hier-Majumder, 2009), k^2

$$\lambda = -\frac{}{\mathcal{P}e}\left(\frac{Wb\phi_o^{-1/2}}{1 - Wb\phi_o^{-1/2}}\right), \tag{2}$$

where $\mathcal{P}e$ is the dimensionless Peclet number, and W and b are constants. These quantities are defined as,

$$\mathcal{P}e = \frac{vL}{D},\ W = \frac{\gamma\Omega}{dE},\ b = \frac{c_0}{2\,(1-c_0)}f(\theta), \tag{3}$$

where $f(\theta)$ is a function of the dihedral angle (Takei and Hier-Majumder, 2009, equation 7), and the definition and values of the remaining constants are described in Table 1. The characteristic velocity of melt is considered to be 1μm/10 hr, a conservative estimate of the velocity of melt segregation, compared to the length of our experiments and the studies of King et al. (2011).

We use the growth rate from equation (2) using the constants listed in the table, to determine the enrichment in the melt fraction defined as,

$$\Delta\phi = \frac{\Delta\phi_t}{\Delta\phi_0}, \tag{4}$$

where the numerator is the difference between maximum melt fraction and minimum melt fraction at time t and the denominator is the difference between maximum and minimum melt fraction at the beginning of the annealing experiment (King et al., 2011).

Variable	Definition	Value
v	velocity of melt	2.78×10^{-11}ms^{-1} *
L	length scale of subvolume	7×10^{-4} m *
D	diffusivity	5×10^{-11} m^{2}s^{-1} [a]
γ	surface tension	1 Jm^{-2}*
Ω	molar volume of Fe	7.09 $\times 10^{-6}$ m^{3}mol^{-1} [a]
d	grain size	$10 - 50$ μm [c]
E	activation energy of reaction	1.2×10^4 Jmol^{-1} [a]
c_0	initial concentration of Fe in the melt	0.7 [c]

Sources: * This study, [a] Takei and Hier-Majumder (2009), [b] Robie and Bethke (1962), [c] Solferino and Golabek (2018).

Table 1. Table of constants used in the calculation of growth rate of linear perturbations.

2 DATA TABLE

This data table contains the total and connected melt fractions and threshold range of the image sub-volumes.

Sample	Total Volume Fraction Melt Distribution	Connected Volume Fraction Melt Distribution	Threshold Range

	(%)	(%)		
FS9010–1	Sub-Volume–1	19.68	19.18	128–255
	Sub-Volume–2	20.09	19.64	128–255
	Sub-Volume–3	20.40	19.99	128–255
	Sub-Volume–4	18.97	18.44	128–255
	Sub-Volume–5	19.13	18.64	128–255
	Sub-Volume–6	19.66	19.18	128–255
FS9010–4	Sub-Volume–1	15.95	7.32	124–255
	Sub-Volume–2	15.47	2.92	124–255
	Sub-Volume–3	16.11	5.61	124–255
	Sub-Volume–4	16.39	6.79	124–255
	Sub-Volume–5	17.20	8.5	124–255
	Sub-Volume–6	16.85	7.34	124–255
FS9010–5	Sub-Volume–1	17.38	7.78	116–255
	Sub-Volume–2	18.84	9.32	116–255
	Sub-Volume–3	17.09	5.95	116–255
	Sub-Volume–4	18.46	6.7	116–255
	Sub-Volume–5	16.88	0.0	116–255
	Sub-Volume–6	16.53	1.84	116–255
OFS–3	Sub-Volume–1	28.32	28.28	113–255
	Sub-Volume–2	22.89	22.86	113–255
	Sub-Volume–3	26.16	26.13	113–255
	Sub-Volume–4	25.75	25.66	113–255
	Sub-Volume–5	21.07	21.02	113–255
	Sub-Volume–6	39.38	39.37	113–255
OFS–4	Sub-Volume–1	3.95	0.0	130–255
	Sub-Volume–2	5.26	0.0	130–255
	Sub-Volume–3	5.45	0.0	130–255
	Sub-Volume–4	4.21	0.0	130–255
	Sub-Volume–5	6.01	0.0	130–255
	Sub-Volume–6	4.92	0.0	130–255
OFS–5	Sub-Volume–1	15.59	15.39	138–255
	Sub-Volume–2	13.81	13.67	138–255
	Sub-Volume–3	14.64	14.54	138–255
	Sub-Volume–4	15.62	15.5	138–255
	Sub-Volume–5	15.70	15.57	138–255
	Sub-Volume–6	15.5	15.41	138–255
OFS–9	Sub-Volume–1	14.96	14.15	85–255
	Sub-Volume–2	24.13	23.78	85–255
	Sub-Volume–3	20.58	20.13	85–255
	Sub-Volume–4	21.46	21.21	85–255
	Sub-Volume–5	20.72	20.43	85–255

	Sub-Volume–6	23.58	23.38	85–255
OFS–10	Sub-Volume–1	20.97	20.85	82–255
	Sub-Volume–2	19.11	18.9	82–255
	Sub-Volume–3	19.92	19.79	82–255
	Sub-Volume–4	21.95	21.92	82–255
	Sub-Volume–5	19.42	19.33	82–255
	Sub-Volume–6	21.92	21.78	82–255
OFS–11	Sub-Volume–1	17.91	14.45	146–255
	Sub-Volume–2	17.43	14.82	146–255
	Sub-Volume–3	18.38	16.62	146–255
	Sub-Volume–4	17.36	15.09	146–255
	Sub-Volume–5	17.73	15.73	146–255
	Sub-Volume–6	18.30	16.07	146–255
OFS–15	Sub-Volume–1	38.43	38.4	132–255
	Sub-Volume–2	38.24	38.2	132–255
	Sub-Volume–3	43.9	43.89	132–255
	Sub-Volume–4	47.48	47.47	132–255
	Sub-Volume–5	45.15	45.13	132–255
	Sub-Volume–6	43.94	43.92	132–255
OFS–16	Sub-Volume–1	29.8	29.79	120–255
	Sub-Volume–2	30.77	30.75	120–255
	Sub-Volume–3	29.66	29.56	120–255
	Sub-Volume–4	31.09	31.04	120–255
	Sub-Volume–5	31.12	31.09	120–255
	Sub-Volume–6	32.06	32.06	120–255

Table 2 Results of melt distribution volume calculation. Segmented using Interactive (manual) thresholding.

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