"Studying Complex Adaptive Systems with Internal States: A Recurrence Network Approach to the Analysis of Multivariate Time Series Representing Self-Reports of Human Experience"

Supplementary Material

1 Change Profile

1.1 Effect of window size on the descriptive statistics of the change profile

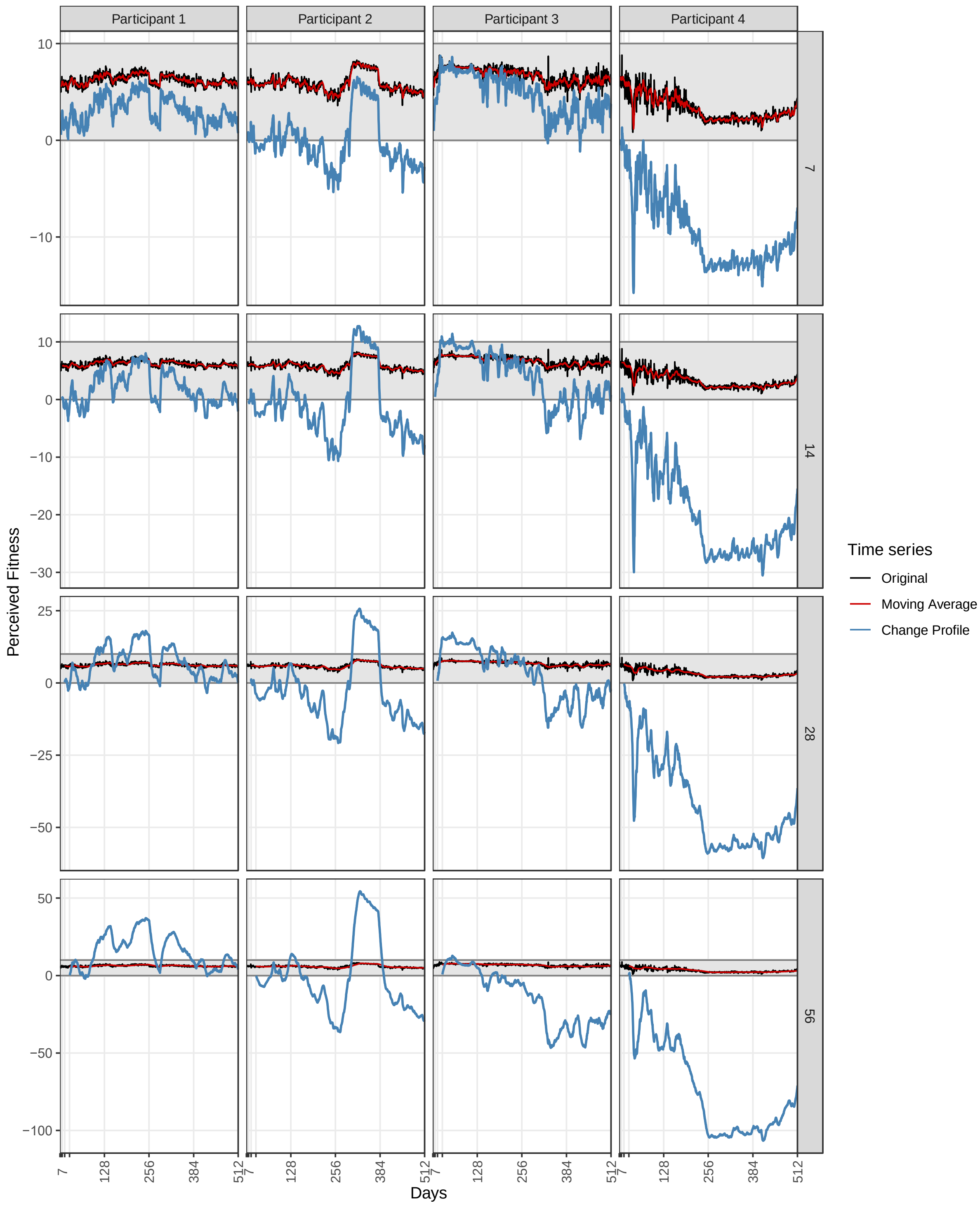
The window size chosen for the rolling average in the calculation of the change profile has two main effects:

1. *The range of the unbounded change profile*: Larger window sizes will in general result in a greater range of values (see Supplementary Figure 1) and this will of course affect the descriptive statistics (see Supplementary Table 1).
2. *Linear filter*: Subtracting a moving average series from a time series resembles smoothing by a linear filter. A larger window size results in more smoothing of high frequency fluctuations (see Supplementary Table 1, e.g. Relative Roughness).

These effects are intended, and should be considered when deciding on a particular window size. The purpose of creating an unbounded profile in the present paper is to make different regimes of self-reporting behavior more salient, under the assumption that the participants will in general not use an absolute interpretation of the rating scale, but one that will change relative earlier interpretation of the scale. The window size should represent a time scale of which it is reasonable to assume that past evaluations within this window of time will serve as a reference level for current self-reports.

1.2 Effect of window size on the recurrence analysis of the change profile

To see the effect of using a change profile on any subsequent recurrence analyses consider participant 4 in Supplementary Figure 1. A clear decrease in the mean level of ratings is visible in the original series. Visual inspection of the change profiles (blue lines in the rows) seem to indicate this concerns a sudden jump around day 256 to a new level, rather than a continuation of the decreasing stationary trend. This new level does not remain stationary, but reveals an upward trend towards the end of the series. This pattern is of course present in the original data, the profile exaggerates these patterns and this is relevant for deciding which values should be considered recurring states in a recurrence matrix. For window sizes of 7 and 14, the low value observed early in the time series is an extreme observation, which can be considered to recur around day 256. For window size 28 and 56, this early low value is not as extreme, and will probably not be considered a state that recurs around day 256. In the present paper we chose to consider these values might represent recurring internal states and chose window size of 14 (7 days).



Supplementary Figure 1. The effect of increasing window size on the change profile. Notice the y-axis scale changes for each row. Descriptive statistics are reported in table S1.

Supplementary Table 1

Descriptive statistics of the original time series, and change profiles with different window sizes for the rolling average.

Participant	Window	mean	SD	median	MAD	minimum	maximum	range	autocovariance lag 1	autocovariance lag 0	Relative Roughness
1	observed	6.24	0.52	6.20	0.59	4.90	7.67	2.77	0.20	0.27	0.5020
1	7	3.12	1.36	2.96	1.56	0.13	6.27	6.14	1.80	1.85	0.0469
1	14	2.01	2.82	1.62	3.21	-3.72	8.04	11.80	7.88	7.94	0.0145
1	28	6.75	5.60	6.07	6.71	-3.45	18.10	21.50	31.20	31.30	0.0068
1	56	15.40	10.60	14.00	12.10	-2.01	37.10	39.10	111	112	0.0048
2	observed	5.88	0.94	5.73	0.79	3.58	8.29	4.71	0.83	0.88	0.1160
2	7	-0.13	2.74	-0.67	2.17	-5.41	6.56	12.00	7.45	7.49	0.0112
2	14	-1.11	5.86	-2.28	4.55	-10.70	12.70	23.40	34.20	34.30	0.0047
2	28	-2.67	12	-4.88	8.67	-20.80	25.70	46.50	143	143	0.0027
2	56	-1.27	23.80	-5.87	18.40	-36.60	54.40	91	566	567	0.0019
3	observed	6.74	0.78	6.85	0.92	4.00	8.80	4.80	0.48	0.61	0.4510
3	7	4.76	2.11	4.92	2.63	-1.16	8.74	9.91	4.36	4.44	0.0352
3	14	3.95	4.43	4.60	5.65	-6.88	11.40	18.30	19.50	19.60	0.0096
3	28	3.26	9.03	5.09	11.60	-15.50	17.40	32.90	81.30	81.40	0.0030
3	56	-14.40	18	-11.10	26.10	-46.70	12.80	59.50	323	323	0.0018
4	observed	3.30	1.36	2.85	1.11	0.83	8.81	7.98	1.65	1.86	0.2200
4	7	-9.40	3.82	-10.70	3.31	-15.80	1.33	17.1	14.40	14.60	0.0239
4	14	-19.60	8.03	-22.50	7.06	-30.50	1.93	32.5	64	64.40	0.0116
4	28	-41.90	15.90	-47.30	14.40	-60.60	-0.18	60.4	250	251	0.0089
4	56	-73.80	30	-84.40	27.80	-107	1.62	108	899	902	0.0069

Note: The relative roughness is a ratio of the local variance in time series X (the autocovariance at lag=1, $K_{XX}(1)$) over the global variance (the autocovariance at lag=0, or simply $var(X)$). $RelativeRoughness = 2 * \left(1 - K_{XX}(1)\right)/var(X)$. See Marmelat, Torre (1) for details.

2 Spiral graph layout

Figure 8 shows several different spiral layouts that can be used to highlight different epochs in the time series. The vertex coordinates for the *Archimedean*, *Bernoulli*, and *Fermat* spirals are generated as follows:

$$x = r * \cos\theta$$
$$y = r * \sin\theta$$

Where θ is a sequence $\{\theta_n\}_{n=1}^{N}$ with range $\{0, \ldots, \pi * N_{arcs}\}$, with N equal to the number of vertices and N_{arcs} the number of arcs in the spiral. Depending on the spiral type, r takes the form:

$$\begin{aligned} &\text{Archimedean:} && r = a + b * \theta \\ &\text{Bernoulli:} && r = a + e^{b*\theta} \\ &\text{Fermat:} && r = a + b * \theta^2 \end{aligned}$$

Where a and b define the starting radius and the distance between successive arcs, respectively.

The double-ended *Euler* spiral is constructed from a clothoid, calculated for half the number of vertices:

$$x = \int_0^{\frac{\pi}{2}} \tfrac{1}{b+1} \cos\theta^{b+1} * d\theta$$

$$y = \int_0^{\frac{\pi}{2}} \tfrac{1}{b+1} \sin\theta^{b+1} * d\theta$$

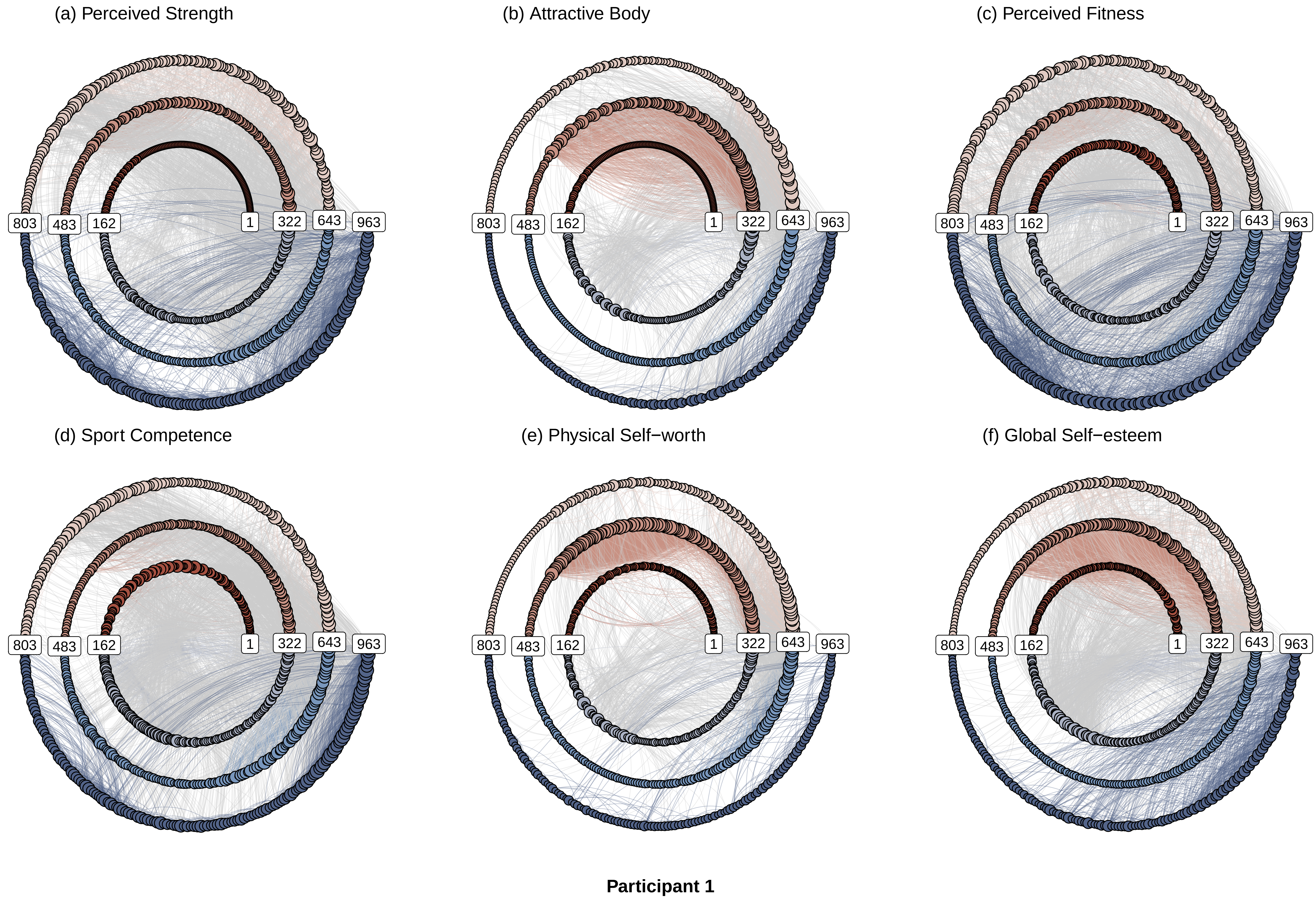
Where θ is a sequence $\{\theta_n\}_{n=1}^{\frac{N}{2}}$with range $\{0, \ldots, \frac{\pi}{2}\}$ and $d\theta = \frac{\theta}{N_{arcs}}$. N and N_{arcs} equal to the number of vertices and the number of arcs in the spiral, respectively. To create the first half of the sequence, the coordinates are reversed and flipped:

$$x = \{-\,reverse(x), x\}$$

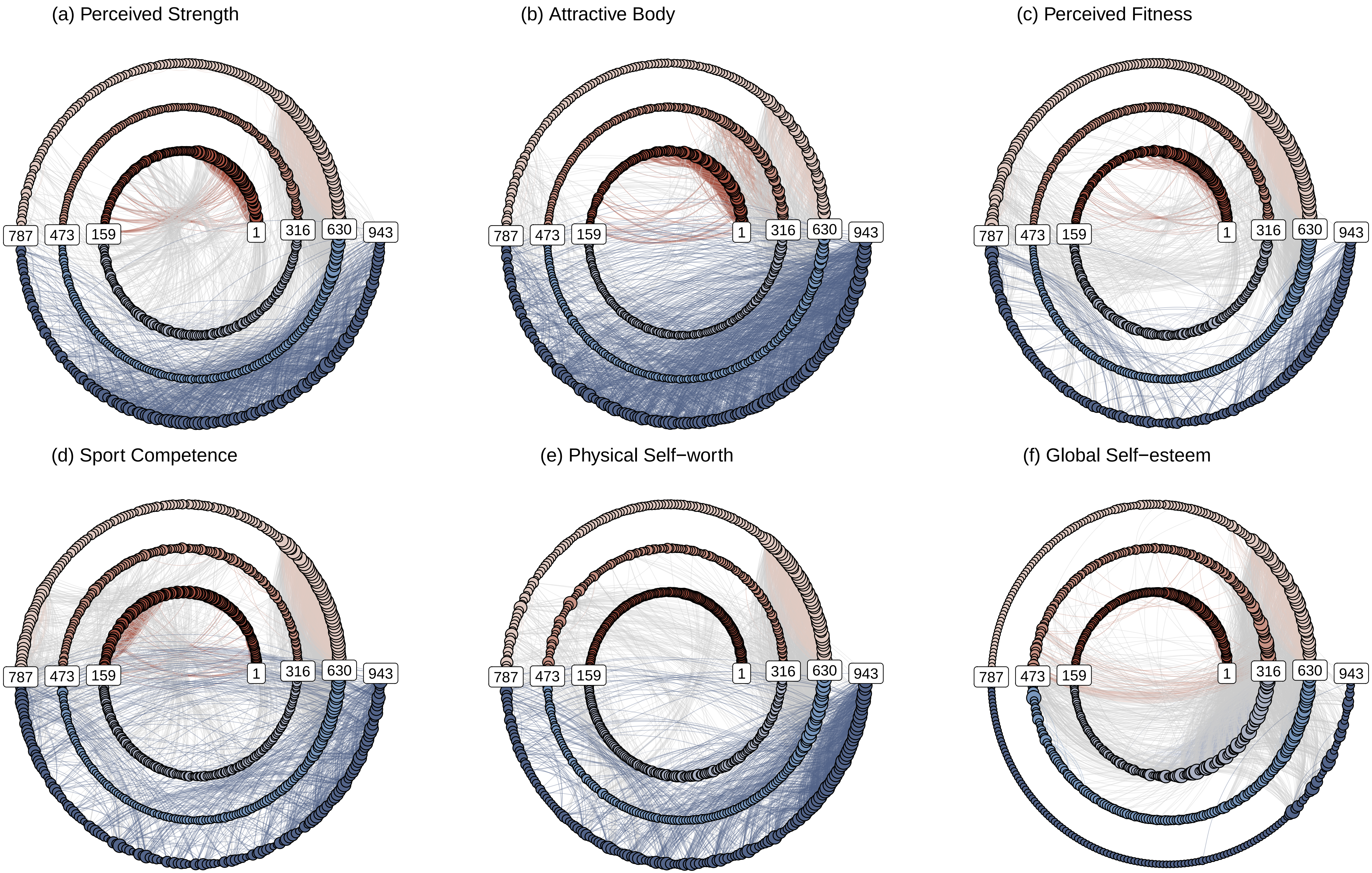
$$y = \{-\,reverse(y), y\}$$

The functions *layout_as_spiral()*, *make_spiral_graph()* in R-package *casnet* [2] can be used to generate the spiral coordinates and figures, for a tutorial see: https://fredhasselman.com/casnet/articles/RecurrenceNetworks.html.

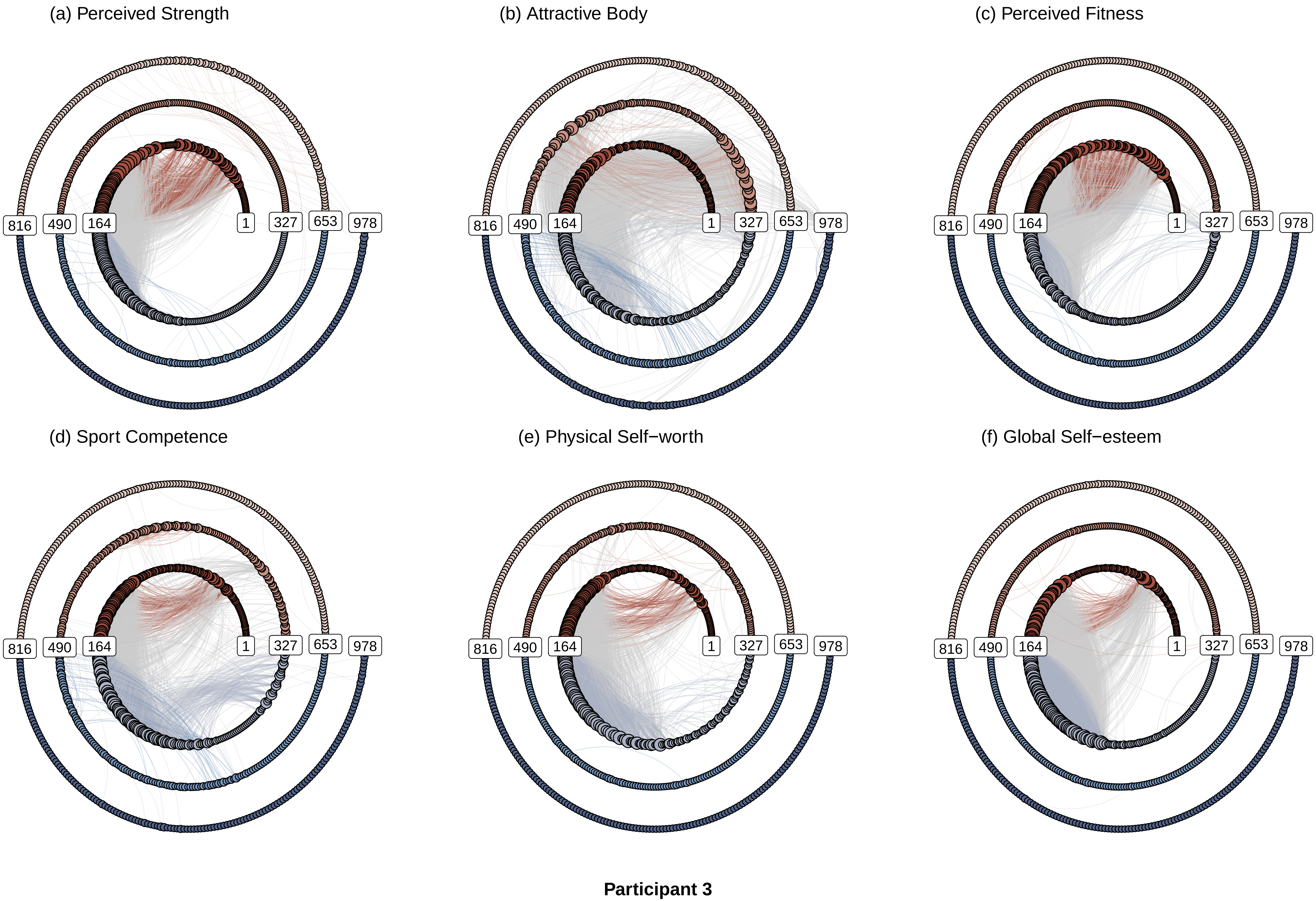
3 Recurrence network graphs of all participants and variables

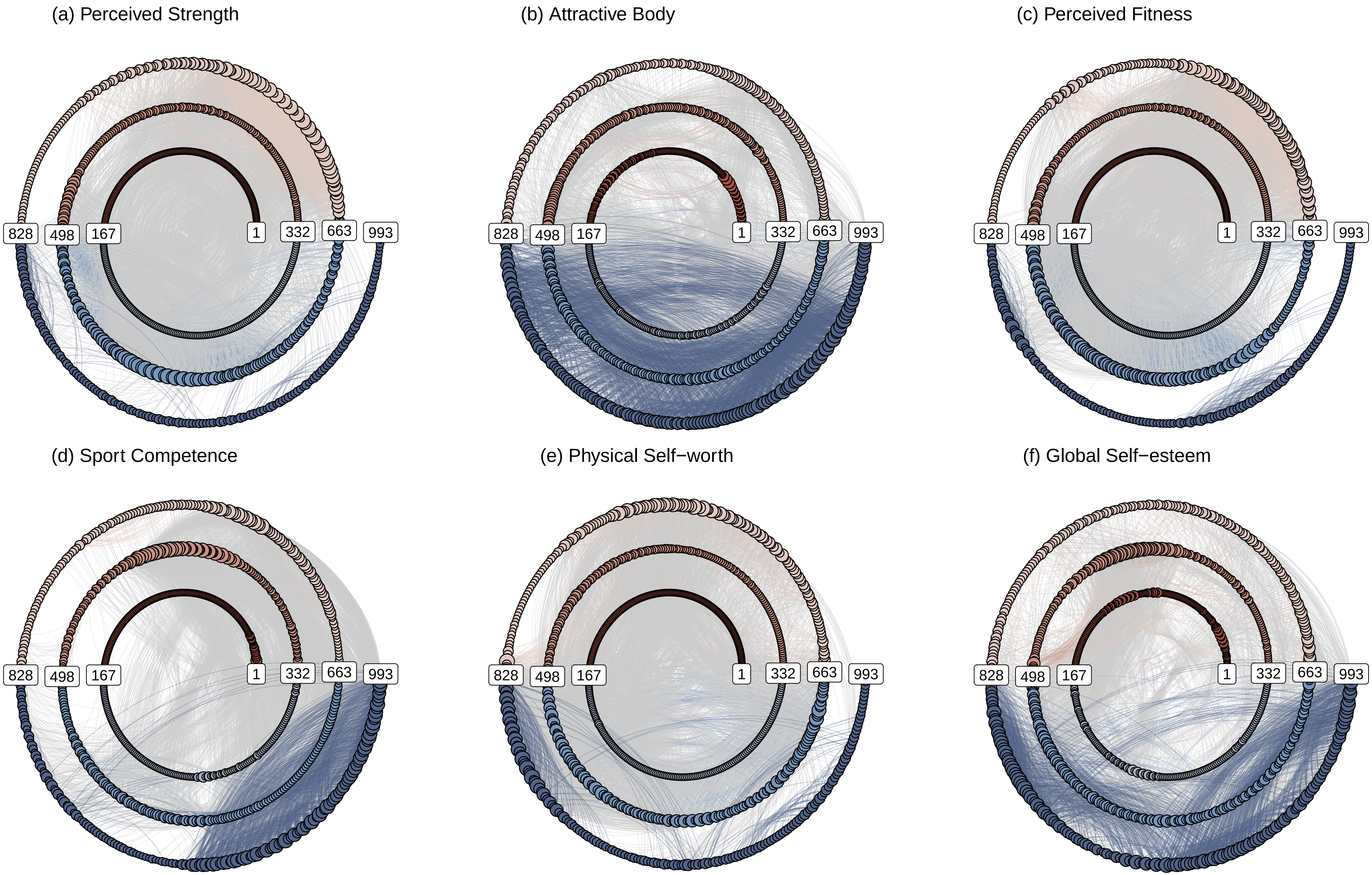


(a) Perceived Strength
(b) Attractive Body
(c) Perceived Fitness
(d) Sport Competence
(e) Physical Self−worth
(f) Global Self−esteem
803
483
162
1
322
643
963
Participant 1



Participant 2


(a) Perceived Strength
(b) Attractive Body
(c) Perceived Fitness
(d) Sport Competence
(e) Physical Self−worth
(f) Global Self−esteem
816
490
164
1
327
653
978
Participant 3



Participant 4

References

1. Marmelat V, Torre K, Delignieres D. Relative roughness: an index for testing the suitability of the monofractal model. Front Physiol. 2012;3:208.
2. Hasselman F. casnet: A toolbox for studying Complex Adaptive Systems and NETworks 0.1.3 ed2017.