A Deep Learning Approach to Predict Abdominal Aortic Aneurysm Expansion using Longitudinal Data

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APPENDIX A

The G&R computational model is used to generate in silico data for pre-training the deep structure. In the G&R, the arterial wall is considered a homogenized mixture of three stress-bearing constituents, including elastin, collagen fiber and smooth muscle. The strain energy for the elastin, collagen fiber families (k =1,...,4) and passive smooth muscle are given as

$$\begin{aligned}
\Psi^e(\mathbf{C}^e_n(t)) &= \frac{c_1}{2}\left(C^e_{[11]} + C^e_{[22]} + \frac{1}{C^e_{[11]}C^e_{[22]} - {C^e_{[12]}}^2} - 3\right),\\
\Psi^c(\lambda^k_{n(\tau)}(t)) &= \frac{c_2}{4c_3}\left\{\exp[\mathrm{c}_3(\lambda^{\mathrm{k}}_{\mathrm{n}(\tau)}(\mathrm{t})^2 - 1)^2] - 1\right\},\\
\Psi^m(\lambda^m_{n(\tau)}(t)) &= \frac{c_4}{4c_5}\left\{\exp[\mathrm{c}_5(\lambda^{\mathrm{m}}_{\mathrm{n}(\tau)}(\mathrm{t})^2 - 1)^2] - 1\right\},
\end{aligned} \tag{1}$$

where $C^e_{[11]}$, $C^e_{[22]}$ and $C^e_{[12]}$ are components of $\mathbf{C}^e_n(t)$ which represents the green tensor of elastin from its stress-free configuration to current configuration; $\lambda^k_{n(\tau)}(t)$ and $\lambda^k_{n(\tau)}(t)$ represent the stretch of collagen fiber from their stress-free configurations to current configurations. $\mathbf{C}^e_n(t)$, $\lambda^k_{n(\tau)}(t)$ and $\lambda^k_{n(\tau)}(t)$ can be provided by kinematics of G&R that is explicitly expressed in Baek et al. (2006); Zeinali-Davarani et al. (2011); Farsad et al. (2015). Accordingly, due to the constant turnover of aortic materials, the total stored energy per unit reference area of AAA is expressed as the summation of all strain energies contributed by various survival masses, i.e.,

$$w(t) = \sum_i \left\{M^i(0)Q^i(t)\Psi(\mathbf{C}^i_{n(0)}(t)) + \int_0^t m^i(\tau)q^i(t,\tau)\Psi(\mathbf{C}^i_{n(\tau)}(t))d\tau\right\}, \tag{2}$$

where $\Psi(\mathbf{C}^i_{n(\tau)}(t))$ represents the stored energy of constituent i synthesized at time τ; $M^i(0)$ represents the initial mass of constituent i; $m^i(\tau)$, which is indicated at M(2) ('M' indicates the equation number referred in the main text), represents the stress-mediated mass production rate of constituent i that synthesized at time τ; $Q^i(t)$ and $q^i(t,\tau)$ represent the survival functions, which are the survival fraction of the initial mass and the mass produced at time τ, respectively.

Along with the given energy function aortic wall $w(t)$, we use the principle of virtual work to provide the weak form of the proposed method, which is expresses as

$$\delta I = \int_S \delta w dA - \int_s P\mathbf{n} \cdot \delta \mathbf{x} da = 0, \tag{3}$$

where $P\mathbf{n}$ denotes the inner pressure vector applied to the aortic wall; S and s correspond to the surface of aorta in reference and current configurations, respectively; $\delta\mathbf{x}$ denotes the virtual changes in position. Using the FEM formulation, we can approximate the current position by

$$\mathbf{x} = \mathbf{\Phi}\mathbf{x}^p, \tag{4}$$

where $\mathbf{x}^p$ is the nodal vector for the current position, and $\mathbf{\Phi}$ is the shape function matrix. Therefore, all the other kinematic variables can be expressed by the shape functions and the current nodal coordinates; thus we obtain the governing equation which can computed using Newton-Raphson method. Details of FEM formulation and solving strategy can be found in Kyriacou et al. (1996); Baek et al. (2007).

APPENDIX B

Proof of Corollary 3.2:

We illustrate the proof for $N = 2$, then the proof for an arbitrary N is straightforward. Let

$$\begin{aligned} g_0 &= 1, \\ g_1 &= ax + b, \\ g_2 &= cx^2 + dx + e. \end{aligned}$$

So, applying M(13), we have

$$\begin{aligned} \int_x \pi(x) g_0(x) g_3(x) dx &= \int_x \pi(x) g_3(x) dx = 0, \\ \int_x \pi(x) g_1(x) g_3(x) dx &= \int_x \pi(x)(ax+b) g_3(x) dx = 0, \\ \int_x \pi(x) g_2(x) g_3(x) dx &= \int_x \pi(x)(cx^2+dx+e) g_3(x) dx = 0. \end{aligned}$$

Rearrange the terms, we have an equivalent system of equations

$$\begin{aligned} &(1+b+e)\int_x \pi(x) g_3(x) dx = 0 \rightarrow \int_x \pi(x) g_3(x) dx = 0, \\ &(a+d)\int_x \pi(x) x g_3(x) dx = 0 \rightarrow \int_x \pi(x) x g_3(x) dx = 0, \\ &(c)\int_x \pi(x) x^2 g_x(x) dx = 0 \rightarrow \int_x \pi(x) x^2 g_3(x) dx = 0. \end{aligned}$$

which is the condition M(12).

APPENDIX C

In a CD-k learning, the gradient of the parameter θ_i is approximated by

$$\begin{aligned}\Delta\theta_i &= -\epsilon\frac{\log P(\mathbf{x})}{\partial\theta_i}\\ &= \epsilon\left(\mathbb{E}_{P(\mathbf{h}|\mathbf{x})}\left[\frac{\partial E(\mathbf{x},\mathbf{h})}{\partial\theta_i}\right] - \mathbb{E}_{P_k(\tilde{\mathbf{x}},\mathbf{h})}\left[\frac{\partial E(\bar{\mathbf{x}},\mathbf{h})}{\partial\theta_i}\right]\right),\end{aligned}$$

where $P_k(\tilde{\mathbf{x}},\mathbf{h})$ is the distribution of the reconstructed visible data $\bar{\mathbf{x}}$ after k steps of Gibbs sampling and ϵ is the learn rate. In this study, we utilize CD-1 that involves one full step of Gibbs sampling, i.e., $P_1(\tilde{\mathbf{x}},\mathbf{h})$. Applying M(6) to M(17), we have the learning update rules for each RBM layer as follows.

$$\begin{aligned}w_{ij} &\leftarrow w_{ij} + \epsilon\left(\mathbb{E}_{P(\mathbf{h}|\mathbf{x})}\left[\frac{h_j x_i}{\sigma_i}\right] - \mathbb{E}_{P_1(\tilde{\mathbf{x}},\mathbf{h})}\left[\frac{h_j x_i}{\sigma_i}\right]\right),\\ c_j &\leftarrow c_j + \epsilon\left(\mathbb{E}_{P(\mathbf{h}|\mathbf{x})}\left[h_j\right] - \mathbb{E}_{P_1(\tilde{\mathbf{x}},\mathbf{h})}\left[h_j\right]\right),\\ a_i &\leftarrow a_i + \epsilon\left(\mathbb{E}_{P(\mathbf{h}|\mathbf{x})}\left[-\frac{v_i - a_i}{\sigma_i^2}\right] - \mathbb{E}_{P_1(\tilde{\mathbf{x}},\mathbf{h})}\left[-\frac{v_i - a_i}{\sigma_i^2}\right]\right),\end{aligned}$$

where

$$\begin{aligned}\mathbb{E}_{P(\mathbf{h}|\mathbf{x})}\left[\frac{h_j x_i}{\sigma_i}\right] &= \frac{1}{N}\sum_{t=1}^{N}\frac{\tilde{h}_j^{[t]}x_i^{[t]}}{\sigma_i},\\ \mathbb{E}_{P_1(\tilde{\mathbf{x}},\mathbf{h})}\left[\frac{h_j x_i}{\sigma_i}\right] &= \frac{1}{N}\sum_{t=1}^{N}\frac{P(h_j^{[t]}=1|\tilde{x}_i^{[t]},\theta)\tilde{x}_i^{[t]}}{\sigma_i},\\ \mathbb{E}_{P(\mathbf{h}|\mathbf{x})}\left[h_j\right] &= \frac{1}{N}\sum_{t=1}^{N}\tilde{h}_j^{[t]},\\ \mathbb{E}_{P_1(\tilde{\mathbf{x}},\mathbf{h})}\left[h_j\right] &= \frac{1}{N}\sum_{t=1}^{N}P(h_j^{[t]}=1|\tilde{x}_i^{[t]},\theta),\\ \mathbb{E}_{P(\mathbf{h}|\mathbf{x})}\left[-\frac{v_i - a_i}{\sigma_i^2}\right] &= \frac{1}{N}\sum_{t=1}^{N}\left(-\frac{x_i^{[t]} - a_i}{\sigma_i^2}\right),\\ \mathbb{E}_{P_1(\tilde{\mathbf{x}},\mathbf{h})}\left[-\frac{v_i - a_i}{\sigma_i^2}\right] &= \frac{1}{N}\sum_{t=1}^{N}\left(-\frac{\tilde{x}_i^{[t]} - a_i}{\sigma_i^2}\right),\end{aligned}$$

where N is number of samples and $\tilde{h}_j$ and $\tilde{x}_i$ are sampled with the distributions in M(18).

REFERENCES

Baek, S., Rajagopal, K. R., Humphrey, J. D., et al. (2006). A theoretical model of enlarging intracranial fusiform aneurysms. *Transactions-ASME Journal Of Biomechanical Engineering* 128, 142

Baek, S., Valentin, A., and Humphrey, J. D. (2007). Biochemomechanics of cerebral vasospasm and its resolution: Ii. constitutive relations and model simulations. *Annals of biomedical engineering* 35, 1498–1509

Farsad, M., Zeinali-Davarani, S., Choi, J., and Baek, S. (2015). Computational growth and remodeling of abdominal aortic aneurysms constrained by the spine. *Journal of Biomechanical Engineering* 137, 091008

Kyriacou, S., Schwab, C., and Humphrey, J. (1996). Finite element analysis of nonlinear orthotropic hyperelastic membranes. *Computational Mechanics* 18, 269–278. doi:10.1007/s004660050146

Zeinali-Davarani, S., Sheidaei, A., and Baek, S. (2011). A finite element model of stress-mediated vascular adaptation: application to abdominal aortic aneurysms. *Computer Methods in Biomechanics and Biomedical Engineering* 14, 803–817