Evolution of a Relativistic Electron Beam for Tracing Magnetospheric Field Lines - Appendices

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Appendix A: Parallel Overlap of Electron Mini-Pulses

As an electron pulse propagates from the satellite to the atmosphere, the mini-pulses will stretch due to energy spread ΔE. If the duty cycle between emission of electron mini-pulses is sufficiently small, or energy spread significantly large, then the bunches may begin to overlap as they near the Earth's atmosphere. Ideally, mini-pulses should remain distinguishable to ground instruments in order to provide temporal resolution of magnetospheric phenomena on the mini-pulse time scale T_{mp}.

In the beam frame, a simple expression for the longitudinal density profile of a train of mini-pulse's is given as,

$$n(z,t) = \sum_{j=-\infty}^{j=\infty} n_0 \exp\left\{-\frac{(z-jh)^2}{2(L_{mp}/2 + v_{\Delta\parallel} t)^2}\right\}, \tag{1}$$

where $j \in \mathbb{Z}$, n_0 is the electron density at the center of the beam mini-pulse and $h = L_{mp}/\tau_{dc}$ is the separation between mini-pulses.

Figure 1a shows the density distribution in the local frame $\hat{\mathbf{z}}$ direction of three mini-pulses just prior to impact with the Earth ($t \approx t_f$) as predicted by Equation 1 and for a simulation initialized with reference beam properties. There is good agreement between the simulation and theory, with a slight discrepancy in mini-pulse spacing. This discrepancy can be explained by considering the effect of magnetic mirroring on each mini-pulse. As

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each mini-pulse nears the Earth, its relative longitudinal velocity with respect to the trailing mini-pulse will reduce due to magnetic mirroring and therefore slightly bunch the train of mini-pulses. Figure 1b shows results from a similar simulation with increased duty cycle resulting in overlapping of the mini-pulses.

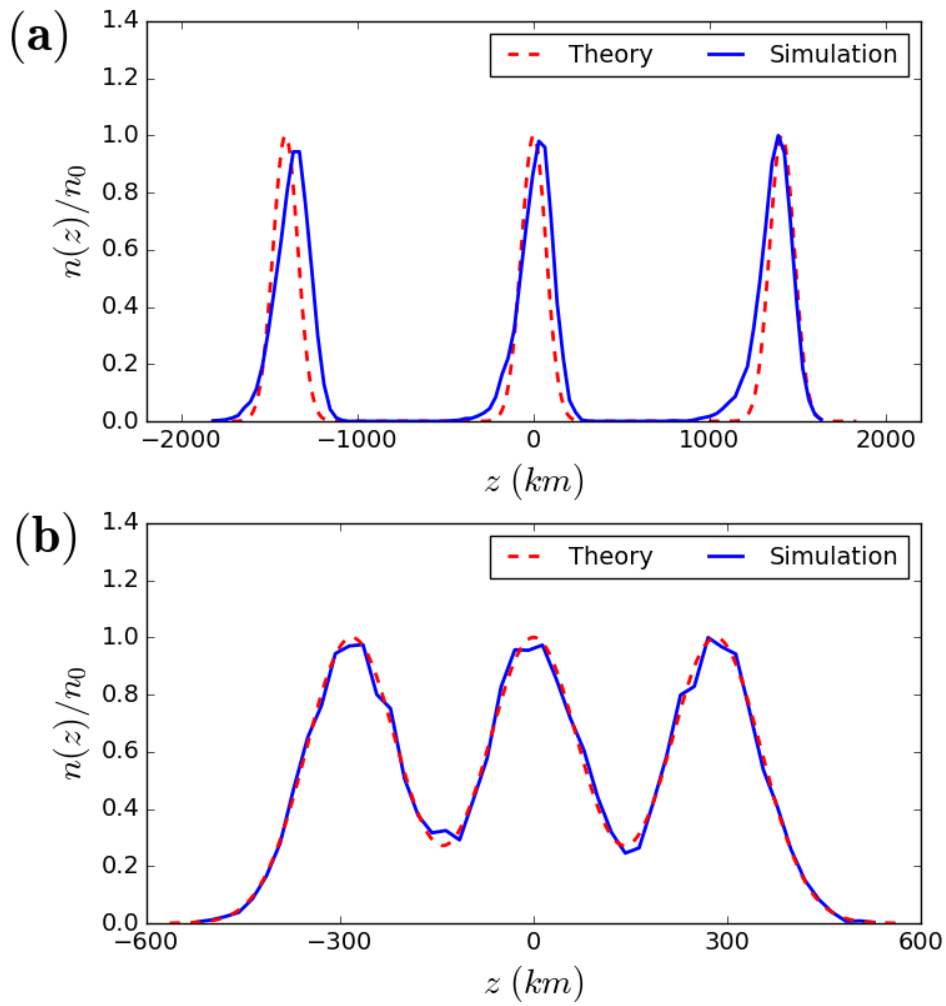

Figure 1: Density distribution of three mini-pulses in the beam frame $\hat{\mathbf{z}}$-direction for (a) $\tau_{dc} = 0.1$ where the bunches remains separate and (b) $\tau_{dc} = 0.5$ where the bunches do overlap.

We define the distinguishability of mini-pulses as the ratio between the maximum to minimum density along the pulse train N_R. The maximum density at any time can be found at the peak of a mini-pulse $z = 0$, and the minimum density can be found at the mid-point between two mini-pulses $z = h/2$. Therefore at impact with the Earth N_R is given by,

$$N_R = \frac{n(0,t_f)}{n(h/2,t_f)} = \sum_{j=-\infty}^{j=\infty} \exp\left\{-\frac{(jh)^2}{2(L_{mp}/2 + v_{\Delta\|}t_f)^2}\right\} \Bigg/ \sum_{j=-\infty}^{j=\infty} \exp\left\{-\frac{(h/2 - jh)^2}{2(L_{mp}/2 + v_{\Delta\|}t_f)^2}\right\} \tag{2}$$

A good approximation for N_R can be obtained by considering only two mini-pulses. With $j \in \{0, 1\}$ Equation 2 becomes,

$$N_R(\Lambda) = \exp\left(-\Lambda^2/4\right)\cosh\left(\Lambda^2/2\right) \tag{3a}$$

$$\Lambda = \frac{L_{mp}/\tau_{dc}}{2(L_{mp}/2 + v_{\Delta\|}t_f)}, \tag{3b}$$

where Λ is a non-dimensional parameter which is a measure of the mini-pulse tendency to overlap, for $\Lambda < 1$ the mini-pulses will be indistinguishable at impact, whereas for $\Lambda > 1$ they will quickly become distinguishable. Figure 2 plots the density ratio N_R against Λ for Equation 2 with $j \in [-100, 100]$ (this was found to be well converged) and Equation 3. Clearly the equations agree very closely up until $N_R \approx 1$ where the peaks and troughs of the pulse train are almost indistinguishable. Therefore Equation 3 can be applied safely to determine mini-pulse distinguishability up until N_R approaches unity, where for higher accuracy Equation 2 must be applied. Figure 1 also shows that for the beam reference conditions $\Lambda \approx 10$ and therefore the mini-pulses will be very clearly distinguished at impact with the Earth.

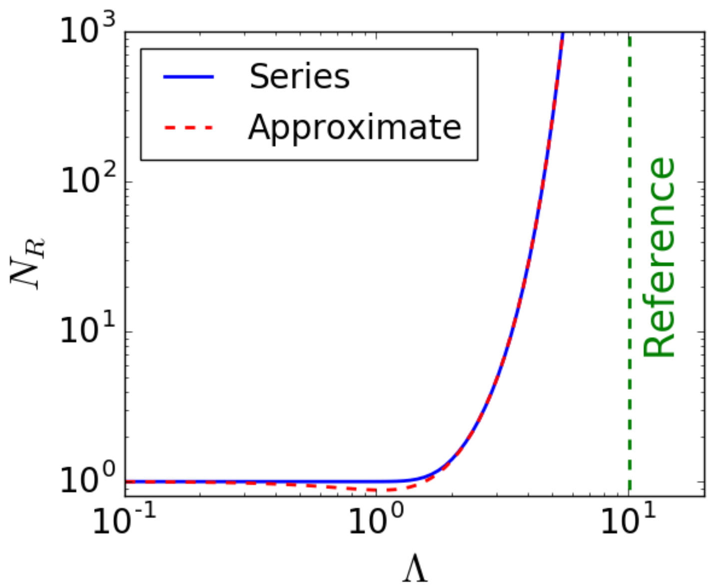

Figure 2: Ratio of maximum to minimum electron density between mini-pulses N_R against beam spreading parameter Λ.

An additional effect not considered by the above analysis is the conversion of initial perpendicular and parallel energy spread, into additional parallel energy spread due to magnetic mirroring. The initial energy spread ΔE and radial emittance ε_r contribute to this effect since they both influence the magnitude of the magnetic moment. Simulations for each of the extreme cases of initial parallel velocity due to energy spread and perpendicular velocity due to emittance were performed and it was found that both of these effects were negligible. Spreading in energy due to mirroring will only be realised as a particle enters the region of steepest magnetic field gradients just before impact with the Earth. Therefore the resulting energy spread due to mirroring will only act over a very small portion of the total particle trajectory and have little overall effect.