# APPENDIX A

In this appendix the optimization approaches, used to determine the characteristics of each device, are presented in order to clarify how the main dynamic parameters of the control devices described in this paper have been chosen to carry out the numerical simulations reported in Sec.3.

The optimization problem is usually formulated as the research of the optimal set of the design variables characterizing the considered passive control device over an admissible domain, chosen so that a defined structural response quantity is minimized. For sake of simplicity, many parameters are supposed fixed by design constraints.

On this base, the main TMDs parameters are certainly the damping ratio and frequency ratio, while other parameters, such as the mass ratio , are supposed determined by practical considerations, thus, their optimization is not generally considered.

In order to avoid onerous calculus, many procedures are usually derived for undamped systems. Therefore, in this work the optimum design of the TMD refers to the explicit formulae developed in (Di Matteo et al., 2019) for systems subjected to a zero-mean stationary Gaussian white noise process. Accordingly to (Di Matteo et al., 2019), the optimum tuning frequency ratio  and damping ratio  of the TMD which can minimize the base isolation sub-system response displacement variance have been derived as:





Equations (A.1 a-b) have been used in order to evaluate the optimal TMD parameters considered in the numerical applications developed in this work.

As far as the New TMD attached to the isolation floor of a structure is concerned, the optimization analysis method described in (Xiang and Nishitani, 2014), initially introduced for fixed base structures and then readapted to the base isolated ones, has been taken into account.

Like traditional TMDs, the only two design variables to be optimized are the frequency ratio  and the damping ratio .

The adopted procedure implies an optimum design method in contrast with the quasi-fixed points theory to obtain wider suppression bandwidths (Xiang and Nishitani, 2014).

Specifically, the authors in (Xiang and Nishitani, 2014) suggested to constrain the values of the parameters so that  and  (Bakre and Jangid, 2007). In particular, since for base-isolated structures it is possible to set the stiffness of the New TMD equal to the stiffness of base isolation bearings (Xiang and Nishitani, 2014), that is , the optimum values of the damping ratio  for different values of mass ratio  can be found considering this condition. In particular, if , the value of  is suggested to be set so that the maximum of the frequency response in terms of base acceleration of the system with the New TMD corresponds to a specific point, where the system equipped with a traditional TMD has a local minimum in the frequency response. Considering the corresponding base acceleration frequency transfer function of the structure with the traditional TMD the position of this point can be determined. As a matter of fact, differentiating the equation of the FRF in terms of base acceleration of the BI + TMD system with respect to the tuning ratio  and then making the obtained equation equal to zero, the coordinates of this point can be derived. In this manner, the New TMD parameters used in the numerical simulations of Sec.3 refer to those reported in (Xiang and Nishitani, 2014).

Dealing with the TLCD, the straightforward estimation of the optimal design parameters proposed in (Di Matteo et al., 2017) is considered. The underlying assumptions concern a simplified equivalent linear system and a zero-mean stationary Gaussian white noise process as base excitation. On this base, a direct optimization procedure for the TLCD device is performed and some charts are supplied as useful tools for a practical design of the TLCD. Further, a comparison with the optimal parameters obtained considering the particle-swarm optimization (PSO) theory (Perez and Behdinan, 2007) proves the reliability of the proposed procedure. Specifically, the optimal couple ( and ) is determined by reaching a minimum of the base-isolation system displacement variance .

Further, assuming that higher powers of  can be neglected , and considering that generally , a direct relationship that provides the equivalent damping ratio  as a function of the input strength  can be obtained as



in which  has been substituted by the term  and .

Therefore, if the base-isolation systems and TLCD parameters are known, Equation (A.2) can be directly used to evaluate  instead of applying computationally more expensive procedures (such as the Statistical Linearization Technique - SLT).

On the other hand, recalling that , an expression relating the head loss coefficient  to the equivalent damping ratio  can be obtained recasting Equation (A.2) as



where



Note that Equations (A.3) and (A.4) are rather useful for a straightforward evaluation of the optimal TLCD design parameters.

On the basis of these considerations and of the formula reported in (Di Matteo et al., 2017) the parameters of the TLCD have been chosen for the numerical simulations reported in Sec.3.