

1 **GLAM Bio-Lith RT: A tool for Remote Sensing Reflectance**
2 **Simulation and Water Components Concentration Retrieval**
3 **in Glacial Lakes**

4 Enrico Schiassi, Department of System and Industrial Engineering, The University of Arizona,
5 Arizona, USA

6 eschiassi@email.arizona.edu

7 Roberto Furfaro *, Department of System and Industrial Engineering, Department of Aerospace
8 and Mechanical Engineering, The University of Arizona, Arizona, USA

9 robertof@email.arizona.edu

10 Jeffrey S. Kargel, Planetary Science Institute, Tucson, Arizona, USA

11 jkargel@psi.edu

12 C. Scott Watson, Department of Hydrology and Atmospheric Science, The University of Arizona,
13 Tucson, Arizona, USA

14 scott@rockyglaciers.co.uk

15 Dan H. Shugar, Water, Sediment, Hazards, and Earth-surface Dynamics (waterSHED) Lab,
16 Department of Geoscience and Environmental Science Program, University of Calgary, Alberta,
17 Canada

18 daniel.shugar@ucalgary.ca

19 Umesh K. Haritashya, Department of Geology, University of Dayton, Dayton, OH, USA

20 uharitashya1@udayton.edu

21 **Appendix**

22 **A Modeling of the Remote Sensing Reflectance Below the**
23 **Surface for Shallow Water**

24 As shown in section 2, the remote sensing reflectance **below** the surface for **shallow water** is
25 modeled as follows:

$$R_{rs}^{sh-}(\lambda) = R_{rs}^-(\lambda) \left[1 - A_{rs,1} e^{-z_B(K_d(\lambda) + k_{uw}(\lambda))} \right] + A_{rs,2} R_{rs}^b(\lambda) e^{-z_B(K_d(\lambda) + k_{uB}(\lambda))} \quad (\text{A.0.0.1})$$

*Corresponding Author

26 where on the right-hand side the first term is the remote sensing reflectance of a water slab with
 27 thickness z_B , and the second term is the remote sensing reflectance of the lakes's bottom seen at
 28 the lake surface within the water column. $A_{rs,1} = 1.1576$ and $A_{rs,2} = 1.0389$ are fixed empirical
 29 constant defined in [7]. $R_{rs}^b(\lambda)$ is the lake bottom reflectance and is calculated as follows:

$$R_{rs}^b(\lambda) = \sum_{n=0}^{N-1} f_n B_n a_n(\lambda) \quad (\text{A.0.0.2})$$

30 In A.0.0.2, six types of bottom surfaces are considered. Hence six albedos (irradiance reflectance of
 31 a surface is called albedo) $a_n(\lambda)$ are considered at the same time. The surface albedos spectra are
 32 provided for six different types of bottom. f_n is the areal fraction of bottom surface type n , and
 33 $\sum_{n=0}^{N-1} f_n = 1$. B_n is the proportion of radiation that is reflected to the sensor. [4] and so do we
 34 consider all the surface types as Lambertian surfaces (isotropic reflection) hence $B_n = 1/\pi = 0.318$
 35 sr^{-1} for all n .

36 K_d , k_{uW} , and k_{uB} measure the radiation attenuation within the water slab (attenuation for down-
 37 welling irradiance, upwelling radiance reflected in the water, and upwelling radiance reflected by
 38 the bottom respectively) and they are computed as below:

$$K_d(\lambda) = k_0 \frac{a(\lambda) + b_b(\lambda)}{\cos\Theta'_{sun}} \quad (\text{A.0.0.3})$$

$$k_{uW}(\lambda) = \frac{a(\lambda) + b_b(\lambda)}{\cos\Theta'_V} [1 + \omega_b(\lambda)]^{3.5421} \left[1 - \frac{0.2786}{\cos\Theta'_{sun}} \right] \quad (\text{A.0.0.4})$$

$$k_{uB}(\lambda) = \frac{a(\lambda) + b_b(\lambda)}{\cos\Theta'_V} [1 + \omega_b(\lambda)]^{2.2658} \left[1 - \frac{0.0577}{\cos\Theta'_{sun}} \right] \quad (\text{A.0.0.5})$$

39 k_0 depends on the scattering phase function. It usually set equal to 1.0395 for case-1 water and
 40 equal to 1.0546 for case-2 water ([4] default).

41 The remote sensing reflectance **above** the water surface can be model in different ways, the following
 42 one is set by default in [4] and used here:

$$R_{rs}(\lambda) = \frac{(1 - \sigma)(1 - \sigma_L^-)}{n_W^2} \cdot \frac{R_{rs}^-(\lambda)}{1 - \rho_u Q R_{rs}^-(\lambda)} + R_{rs}^{surf}(\lambda) \quad (\text{A.0.0.6})$$

43 On the right-hand side, the first term measures the reflection in the water, the second at the
 44 surface. $n_W = 1.33$ is the water index of refraction. $Q = 5 \text{ sr}$ is defined in the previous sections.
 45 σ is the reflection factor for the downwelling irradiance above the water surface. It is set equal to
 46 0.03 by default. σ_L^- is the reflection factor for the upwelling radiance below the water surface. It is
 47 computed as follows:

$$\sigma_L^-(\Theta_v) = \rho_L(\Theta_v) = \frac{1}{2} \left| \frac{\sin^2(\Theta_v - \Theta'_v)}{\sin^2(\Theta_v + \Theta'_v)} + \frac{\tan^2(\Theta_v - \Theta'_v)}{\tan^2(\Theta_v + \Theta'_v)} \right| \quad (\text{A.0.0.7})$$

48 where the viewing angles are related by the Snell's law:

$$n_W \sin \Theta'_v = n_{air} \sin \Theta_v \quad (\text{A.0.0.8})$$

49 ρ_u is the reflection factor for the upwelling irradiance below the water surface. It is set equal to
 50 0.54 by default. $R_{rs}^-(\lambda)$ is the remote sensing reflectance below the surface for deep water described
 51 previously. If shallow water is considered, $R_{rs}^-(\lambda)$ is replaced with $R_{rs}^{sh-}(\lambda)$, which is described
 52 above as well.

53 $R_{rs}^{surf}(\lambda)$ is the reflection at the surface, also called specular reflectance, and is modeled as follows:

$$R_{rs}^{surf}(\lambda) = \rho_L \frac{L_s(\lambda)}{E_d(\lambda)} \quad (\text{A.0.0.9})$$

54 If the wavelength-independent model is selected A.0.0.9 is replaced by:

$$R_{rs}^{surf} = \frac{\rho_L}{\pi} \quad (\text{A.0.0.10})$$

55 ρ_L is called the Fresnel reflectance and is given by A.0.0.7. $L_s(\lambda)$ and $E_d(\lambda)$ are the sky radiance
 56 and the downwelling irradiance above the water surface, respectively. In our software, we modeled
 57 these two quantities as in [4]. For the sake of completeness, they are reproduced here with a brief
 58 explanation.

59 The downwelling irradiance $E_d(\lambda)$ is modeled as follows:

$$E_d(\lambda) = f_{dd} E_{dd}(\lambda) + f_{ds} E_{ds}(\lambda) \quad (\text{A.0.0.11})$$

60 $E_{dd}(\lambda)$ is the direct component of $E_d(\lambda)$. It represents the sun disk in the sky as photon source.
 61 $E_{ds}(\lambda)$ is the component of $E_d(\lambda)$ scattered from the sky and is the sum of two components:

$$E_{ds}(\lambda) = E_{dsr}(\lambda) + E_{dsa}(\lambda) \quad (\text{A.0.0.12})$$

62 $E_{dsr}(\lambda)$ measures the Rayleigh scattering. $E_{dsa}(\lambda)$ measures the aerosol scattering. f_{dd} and f_{ds} are
 63 correction factors that correct the intensities of the light sources according to the illumination con-
 64 ditions. They allow simulation and measurements analysis at non-standard illumination conditions
 65 [4]. $E_{dd}(\lambda)$, $E_{dsr}(\lambda)$, and $E_{dsa}(\lambda)$ are calculated in the following way:

$$E_{dd}(\lambda) = E_0(\lambda) T_r(\lambda) T_{aa}(\lambda) T_{as}(\lambda) T_{oz}(\lambda) T_o(\lambda) T_{wv}(\lambda) \cos \Theta_{sun} \quad (\text{A.0.0.13})$$

$$E_{dsr}(\lambda) = \frac{1}{2} E_0(\lambda) (1 - T_r^{0.95}(\lambda)) T_{aa}(\lambda) T_{as}(\lambda) T_{oz}(\lambda) T_o(\lambda) T_{wv}(\lambda) \cos \Theta_{sun} \quad (\text{A.0.0.14})$$

$$E_{dsa}(\lambda) = E_0(\lambda) T_r^{1.5}(\lambda) T_{aa}(\lambda) (1 - T_{as}(\lambda)) T_{oz}(\lambda) T_o(\lambda) T_{wv}(\lambda) \cos \Theta_{sun} F_a \quad (\text{A.0.0.15})$$

66 $\cos\Theta_{sun}$ is the solar zenith angle. $E_0(\lambda)$ is the solar irradiance coming from the sun. It is corrected
 67 for orbital eccentricity and sun-earth distance. It is imported from the database. F_a is the aerosol
 68 forward scattering probability and is modeled as follows:

$$F_a = 1 - 0.5e^{[(B_1+B_2\cos\Theta_{sun})\cos\Theta]} \quad (\text{A.0.0.16})$$

69 where:

$$B_1 = B_3 [1.459 + B_3 (0.1595 + 0.4129B_3)] \quad (\text{A.0.0.17})$$

$$B_2 = B_3 [0.0783 + B_3 (-0.3824 - 0.5874B_3)] \quad (\text{A.0.0.18})$$

$$B_3 = -0.1417\alpha + 0.82 \quad (\text{A.0.0.19})$$

70 where α is the Angstrom exponent and ranges from 0.2 to 2 [4].

71 $T_r(\lambda)$, $T_{aa}(\lambda)$, $T_{as}(\lambda)$, $T_{oz}(\lambda)$, $T_o(\lambda)$, and $T_{wv}(\lambda)$ are the transmittance of the atmosphere after
 72 Rayleigh scattering, aerosol absorption, aerosol scattering, ozone absorption, oxygen absorption,
 73 and water vapour absorption respectively. Their equations follow:

$$T_r(\lambda) = e^{\frac{-M'}{115.640\lambda^4 - 1.335\lambda^2}} \quad (\text{A.0.0.20})$$

$$T_{aa}(\lambda) = e^{-(1-\omega_a)\tau_a(\lambda)M} \quad (\text{A.0.0.21})$$

$$T_{as}(\lambda) = e^{-\omega_a\tau_a(\lambda)M} \quad (\text{A.0.0.22})$$

$$T_{oz}(\lambda) = e^{-a_{oz}(\lambda)H_{oz}M_{oz}} \quad (\text{A.0.0.23})$$

$$T_o(\lambda) = e^{\frac{-1.41a_o(\lambda)M'}{[1+118.3a_o(\lambda)M']^{0.45}}} \quad (\text{A.0.0.24})$$

$$T_{wv}(\lambda) = e^{\frac{-0.2385a_{wv}(\lambda)\cdot WV\cdot M}{[1+20.07a_{wv}(\lambda)\cdot WV\cdot M]^{0.45}}} \quad (\text{A.0.0.25})$$

74 M' is the atmospheric path length corrected for nonstandard atmospheric pressure P :

$$M' = \frac{M \cdot P}{1013.25} \quad (\text{A.0.0.26})$$

75 M is the atmospheric path length:

$$M = \frac{1}{\cos\Theta_{sun} + a(90^\circ + b - \Theta_{sun})^{-c}} \quad (\text{A.0.0.27})$$

76 where $a = 0.50572$, $b = 6.079975^\circ$, and $c = 1.253$ [4].

77 M_{oz} is the ozone atmospheric path length:

$$M_{oz} = \frac{1.0035}{(\cos^2\Theta_{sun} + 0.007)^{0.5}} \quad (\text{A.0.0.28})$$

78 ω_a is the aerosol single scattering albedo:

$$\omega_a = (-0.0032AM + 0.972)e^{3.06 \cdot 10^{-4}RH} \quad (\text{A.0.0.29})$$

79 where AM is the air mass type, and RH is the relative humidity. AM ranges from 1 (for open-ocean
80 aerosols) to 10 (continental water aerosols). RH ranges from 46 to 91%.

81 $\tau_a(\lambda)$ is the aerosol optical thickness:

$$\tau_a(\lambda) = \beta \left(\frac{\lambda}{\lambda_a} \right)^{-\alpha} \quad (\text{A.0.0.30})$$

82 α is the Angstrom exponent defined above. $\lambda_a = 550$ nm is the reference wavelength [4]. β is the
83 turbidity coefficient and is modeled as follows:

$$\beta = \tau_a(550) = 3.91 \frac{H_a}{V} \quad (\text{A.0.0.31})$$

84 where V is the horizontal visibility ranging from 8 to 24 km, and H_a is the aerosol scale height
85 which is set equal to 1 km.

86 WV is the water vapor concentration and ranges from 0 to 5 cm.

87 $a_{oz}(\lambda)$, $a_o(\lambda)$, and $a_{wv}(\lambda)$ are the absorption spectra of ozone, oxygen, and water vapour respectively
88 and they are imported from file.

89 The sky radiance $L_s(\lambda)$ is parameterized as follows:

$$L_s(\lambda) = g_{dd}E_{dd}(\lambda) + g_{dsr}E_{dsr}(\lambda) + g_{dsa}E_{dsa}(\lambda) \quad (\text{A.0.0.32})$$

90 where the downwelling irradiances are given above, and $g_{dd} = 0.02$, $g_{dsr} = \frac{1}{\pi}$, and $g_{dsa} = \frac{1}{\pi}$ are the
91 irradiances intensities in sr^{-1} .

92 As stated above the RT model used in GLAM BioLith-RT is the same used in [4, 7]. Thus more
93 details are available in those references.

94 B GLAM BioLith-RT: software implementation and features

95 GLAM BioLith-RT is entirely coded in `Matlab` and available at [https://github.com/nsidc/HMA_GLAM_BioLith-](https://github.com/nsidc/HMA_GLAM_BioLith-RT_5)
96 `RT_5`. All the detailed information on how to run the code are given in the `Readme.txt` file provided
97 with the code. All the input spectra to run the simulations are taken from the database in the folder
98 `DATA` available in WASI4 package <http://www.ioccg.org/data/software.html>. For the constrained
99 optimization framework the `Matlab` function `fmincon` is used. For the Bayesian inversion frame-
100 work, the `MCMC toolbox` developed for `Matlab` available at <http://helios.fmi.fi/ainema/mcmc/> is
101 used.

102 In the GLAM BioLith-RT package, the user finds several data sets, functions, and scripts to compute
103 the quantities of interest. The most important are the following:

- 104 • script `main.m` for:
 - 105 – $R_{rs}(\lambda)$ simulation via the function `AOP_Rrs.m` given the input select by the user
 - 106 – Water component concentrations retrieval given the observed and the simulated $R_{rs}(\lambda)$:
107 constrained optimization framework via the function `InvModelBioLithRT_Copt.m` and
108 Bayesian inversion framework via the function `InvModelBioLithRT_Bopt.m`
- 109 • function `AOP_Rrs.m` for $R_{rs}(\lambda)$ simulation
- 110 • function `InvModelBioLithRT_Copt.m`: objective function for the constrained optimization to
111 concentrations retrieval
- 112 • function `InvModelBioLithRT_Bopt.m`: objective function for the Bayesian inversion to con-
113 centrations retrieval

114 It is up to the user whether to use the software only for $R_{rs}(\lambda)$ simulation (forward modeling mode
115 only) or to retrieve water component concentrations (inverse modeling mode). The inverse modeling
116 mode is the default option. To switch only to the forward modeling mode, the instructions are given
117 below.

118 Inverse modeling mode: inputs

119 Following is the list of the input for the inverse modeling mode:

- 120 • Observed remote sensing reflectance at different wavelengths. The wavelength range allowed
121 is from 400 to 700 nm (visible). Within the allowed range, the user can enter any wavelength
122 desired
- 123 • Case water selection. The user can choose to work with either case-1 or case-2 water
- 124 • View (camera) and Sun angle relative to the zenith, in degrees
- 125 • Water component concentrations (assumed to be constant for the depth of the water slab):
126 phytoplankton (ph), colored dissolved organic matter (CDOM) and suspended particle matter
127 (SPM)
- 128 • Suspended particle matter grain size (default 33.6 μm)
- 129 • Selection between deep water or shallow water (when the bottom contribution is not negligible)

- 130 • Bottom depth and the areal fraction of bottom surface, when shallow water is selected
- 131 • Quantities related to the remote sensing reflectance above the water surface: irradiance inten-
- 132 sities, intensities of light sources, Angstrom exponent, atmospheric pressure, relative humidity,
- 133 scale height for ozone, scale height of the precipitable water in the atmosphere.

134 The default input to run the Bayesian inversion are: number of parameters to be tuned, the tuned
135 parameters first guess to start the MCMC sampling process, prior distributions for the fit param-
136 eters, likelihood function, initial error variance, number of simulations for the MCMC, and the
137 algorithm to perform the MCMC sampling. More inputs are available, and the default settings
138 can be changed, detailed information are available at [http://helios.fmi.fi/lainema/mcmc/mcmc-](http://helios.fmi.fi/lainema/mcmc/mcmc-crun.html)
139 [crun.html](http://helios.fmi.fi/lainema/mcmc/mcmc-crun.html).

140 The wavelength is the predictor/control variable. By default the fit parameters are the water com-
141 ponent concentrations and all the other parameters listed are considered as fixed in the optimization
142 problem. However, the user can change the fit and fixed parameters according to the knowledge
143 that he/she has about the water system of interest. That is, to obtain accurate results in the inverse
144 modeling the uncertainty on the true values of the fixed parameters should be as low as possible.
145 In the following section, a sensitivity analysis is presented to show how the inverse modeling results
146 are sensitive to the uncertainty in the true value of the fixed parameters. The accuracy of the
147 retrieval decreases as the uncertainty in the true value of the fixed parameters increases.

148 **Inverse modeling mode: default outputs**

149 Following is the list of the default outputs for the inverse modeling mode:

- 150 • Simulated spectral remote sensing reflectance (Rrs0) given the water component concentra-
- 151 tions estimated by the user
- 152 • Objective function value (Res0) computed with Rrs0 versus the observed remote sensing
- 153 reflectance Rrs_obs
- 154 • Retrieved concentrations and corresponding objective function value computed via the con-
- 155 strained optimization framework
- 156 • Simulated R_{rs} (Rrs_fit) given the retrieved water component concentrations via the con-
- 157 strained optimization framework
- 158 • Retrieved concentrations probability distributions computed via the Bayesian inversion frame-
- 159 work
- 160 • Simulated R_{rs} (Rrs_B) given the mean value of the retrieved water component concentrations
- 161 probability distributions
- 162 • Plot Rrs0, Rrs_fit, Rrs_B and Rrd_obs versus wavelengths

163 **Forward modeling mode: for spectral remote sensing reflectance simulations**

164 If the user is only interested in spectral remote sensing reflectance simulations, then the option to
165 use the forward mode only should be selected. How to switch to forward mode only is explained
166 in detail in the `Readme.txt`. The inputs for the forward modeling mode are the same as for the
167 inverse modeling mode except for the wavelengths range of interest that in this case needs to be

168 specified. The wavelength range allowed is from 400 to 700 nm (visible). Within the allowed range,
169 the user can enter any wavelength desired. The outputs are the simulated spectral remote sensing
170 reflectance Rrs0 and its plot versus the wavelengths.

171 References

- 172 [1] Claudia Giardino, Gabriele Candiani, Mariano Bresciani, Zhongping Lee, Stefano Gagliano,
173 Monica Pepe (2011), BOMBER: A tool for estimating water quality and bottom properties
174 from remote sensing images, *Computers and Geosciences* 45 (2012) 313-318
- 175 [2] Claudia Giardino, Mariano Bresciani, Emiliana Valentini, Luca Gasperini, Rossano Bolpagni,
176 Vittorio E. Brando (2014) Airborne hyperspectral data to assess suspended particulate matter
177 and aquatic vegetation in a shallow and turbid lake, *Remote Sensing of Environment* 157 (2015)
178 48-57
- 179 [3] Peter Gege (2013) WASI-2D: A software tool for regionally optimized analysis of imaging spec-
180 trometer data from deep and shallow waters, *Computers and Geosciences* 62 (2014) 208–215
- 181 [4] Peter Gege (2015) The Water Color Simulator WASI, User manual for WASI version 4.1
182 <http://www.ioccg.org/data/software.html>
- 183 [5] Zhongping Lee, Kendall L. Carder, Curtis D. Mobley, Robert G. Steward, and Jennifer S.
184 Patch (1998), Hyperspectral remote sensing for shallow waters. 1. A semianalytical model,
185 Optical Society of America, *APPLIED OPTICS* Vol. 37, No. 27, 20 September 1998
- 186 [6] Zhongping Lee, Kendall L. Carder, Curtis D. Mobley, Robert G. Steward, and Jennifer S.
187 Patch (1999), Hyperspectral remote sensing for shallow waters: 2. Deriving bottom depths
188 and water properties by optimization, Optical Society of America, *APPLIED OPTICS*, Vol.
189 38, No. 18, 20 June 1999
- 190 [7] A. Albert, C.D. Mobley (2003), An analytical model for subsurface irradiance and remote
191 sensing reflectance in deep and shallow case-2 waters, Optical Society of America, 3 November
192 2003, Vol. 11, No. 22, *OPTICS EXPRESS* 2874
- 193 [8] Enrico Schiassi, Roberto Furfaro, and Domiziano Mostacci (2016), Bayesian inversion of cou-
194 pled radiative and heat transfer models for asteroid regoliths and lakes, *Radiation Effects and*
195 *Defects in Solids*, VOL. 171, NOS. 9–10, 736–745, DOI: 10.1080/10420150.2016.1253091
- 196 [9] Richard C. Aster, Brian Borchers, and Clifford H. Thurber, *Parameter Estimation and Inverse*
197 *Problems*, Second Edition, DOI: 10.1016/B978-0-12-385048-5.00011-2, 2013 Elsevier Inc.
- 198 [10] Ville Kolehmainen, *Introduction to Bayesian methods in inverse problems*, Department of
199 Applied Physics, University of Eastern Finland, Kuopio, Finland, March 4, 2013, Manchester,
200 UK.
- 201 [11] Heikki Haario, Marko Laine, Antonietta Mira, Eero Saksman *DRAM: Efficient adaptive*
202 *MCMC*, June 2006, *Stat Comput* (2006) 16:339–354, DOI 10.1007/s11222-006-9438-0
- 203 [12] Simon Rogers, Mark Girolami, *A First Course in Machine Learning*, Second Edition, Chapman
204 and Hall/CRC, Machine Learning and Pattern Recognition Series

- 205 [13] Kimes DS, Knyazikhin YP, Privette JL, Abuelgasim AA, Gao F. , Inversion methods for
206 physically-based models. *Remote Sensing Reviews*,2000 Sep 1;18(2-4):381-439.
- 207 [14] Ritchie, J.C. Zimba, P.V. and Everitt, J.H. 2003. Remote Sensing Techniques to As-
208 sess Water Quality. *Photogrammetric Engineering & Remote Sensing*. 69(6), 695-704.
209 10.14358/PERS.69.6.695.
- 210 [15] Dornhofer, K. and Oppelt, N. 2016. Remote sensing for lake research and monitoring – Recent
211 advances. *Ecological Indicators*. 64, 105-122. <http://dx.doi.org/10.1016/j.ecolind.2015.12.009>.
- 212 [16] Chikita, K. Jha, J. and Yamada, T. 1999. Hydrodynamics of a supraglacial lake and its effect
213 on the basin expansion: Tsho Rolpa, Rolwaling Valley, Nepal Himalaya. *Arctic Antarctic and*
214 *Alpine Research*. 31(1), 58-70. 10.2307/1552623.
- 215 [17] Wessels, R.L. Kargel, J.S. and Kieffer, H.H. 2002. ASTER measurement of supraglacial
216 lakes in the Mount Everest region of the Himalaya. *Annals of Glaciology*. 34, 399-408.
217 10.3189/172756402781817545.
- 218 [18] Giardino, C. Oggioni, A. Bresciani, M. and Yan, H. 2010. Remote Sensing of Suspended
219 Particulate Matter in Himalayan Lakes. *Mountain Research and Development*. 30(2), 157-168.
220 10.1659/MRD-JOURNAL-D-09-00042.1.
- 221 [19] Watson, C.S. Quincey, D.J. Carrivick, J.L. Smith, M.W. Rowan, A.V. and Richardson, R.
222 2017. Heterogeneous water storage and thermal regime of supraglacial ponds on debris-covered
223 glaciers. *Earth Surface Processes and Landforms*. 229-241. <http://dx.doi.org/10.1002/esp.4236>.
- 224 [20] Alessandro Ludovisi and Elda GAINO, 2010, Meteorological and water quality changes in Lake
225 Trasimeno (Umbria, Italy) during the last fifty years, *J. Limnol.*, 69(1): 174-188, 2010 DOI:
226 10.3274/JL10-69-1-16
- 227 [21] Marchegiano, M., Francke, A., Gliozzi, E., Wagner, B., & Ariztegui, D. (2019). High-resolution
228 palaeohydrological reconstruction of central Italy during the Holocene. *The Holocene*, 29(3),
229 481–492. <https://doi.org/10.1177/0959683618816465>
- 230 [22] Babin M, Stramski D, Ferrari GM, Claustre H, Bricaud A, Obolensky G, Hoepffner N. 2003.
231 Variations in the light absorption coefficients of phytoplankton, nonalgal particles, and dis-
232 solved organic matter in coastal waters around Europe. *Journal of Geophysical Research*
233 108:3211:4.1–4.20
- 234 [23] Sergios Theodoridis, *Machine Learning, A Bayesian and Optimization Perspective*, First Edi-
235 tion, Academic Press, 2015