Directed flow of information in chimera states: Appendix

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Appendix A: Stability diagram for chimera states

For the study of mutual information, we restricted ourselves to dynamic states most similar to the 'stable chimera states' reported in [1]. Due to the various noise sources inherent to the model, the system dynamics undergoes fluctuations, as described in Section II. This poses additional challenges that we addressed by examining specific time averages of the complex-valued order parameter in each subpopulation. Since fluctuations may cause the system to temporarily drift off the synchronized manifold, we may not simply consider $|Z_1| = 1, |Z_2| < 1$ to identify chimera states. Instead we require the order to be sufficiently different between the two subpopulations, i.e., we threshold $\langle|Z_2|\rangle_t/\langle|Z_1|\rangle_t$, defining the region R_1 shown in Fig. A1(a).

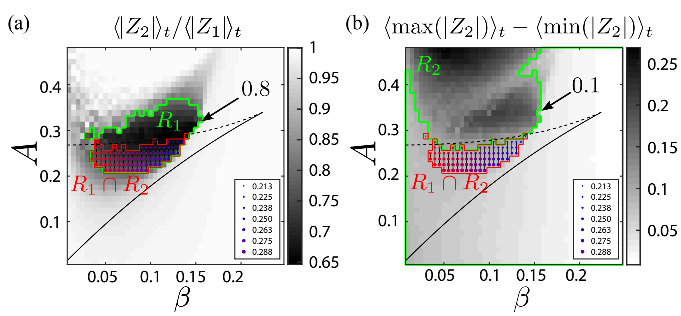

FIG. A1. Stability diagram based on analyzing complex-valued order parameters. Regions R_1 (a) and R_2 (b) in green outline parameter values (β, A) for which $\langle|Z_2|\rangle_t/\langle|Z_1|\rangle_t < 0.8$ and $\langle\max(|Z_2|)\rangle_t - \langle\min(|Z_2|)\rangle_t < 0.1$, respectively. The states that are most similar to 'stable chimera states' [1] were identified within the intersection $R_1 \cap R_2$ (outlined in red). The outlined regions are bounded by parameter regions known to exhibit stable chimeras in the limit of $\sigma_\omega = \sigma_C = \sigma_W^2 = 0$ [1]; saddle node and Hopf bifurcation curves are shown as black solid and dashed curves, respectively. The other parameters have been fixed at $\sigma_\omega = 0.01$, $\sigma_C = 0.01$, and $\sigma_W^2 = 0.001$.

Furthermore, we presume amplitude variations to be small. To that end, we thresholded $\langle\max(|Z_2|)\rangle_t - \langle\min(|Z_2|)\rangle_t$, defining the region R_2 shown in Fig. A1(b). Here, $\langle\cdot\rangle_t$ denotes the average defined over the maxima computed over time windows of length $T = 10^3$ time steps, over a duration $T = 9 \cdot 10^5$ after removing a transient of $T = 10^5$ time steps. Finally, chimera states with parameter values $(\beta, A) \in R_1 \cap R_2$ were selected for further analysis of mutual information.

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Appendix B: Normalization factor $I^{\rm SS}_{\rm max}$

Our model displays a baseline dependency of the maximum peak of mutual information $I^{\rm SS}_{\rm max}$ on the parameters β and A, even when the system is in the fully synchronized state SS, see Fig. A2. We therefore studied the dependency of the maximum peak of the delayed mutual information for chimera states normalized with $I^{\rm SS}_{\rm max}$, $I_{\rm max}/I^{\rm SS}_{\rm max}$. For every set of parameters we computed the delayed mutual information on two different solution that have been forced by changing the initial conditions; the maxima of each of these solutions (SS and SD) are $I^{\rm SS}_{\rm max} \coloneqq I^{\rm SS}_{\rm max}(A, \beta)$ and $I_{\rm max} \coloneqq I^{\rm SD}_{\rm max}(A, \beta)$. These are the quantities displayed in panels (c) and (d) of Fig. 3.

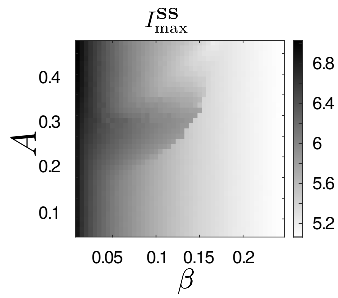

FIG. A2. The normalization factor given by the peak value of mutual information in a fully synchronized state SS, $I^{\rm SS}_{\rm max} \coloneqq I_{11}(\tau_{\rm max}) = I_{22}(\tau_{\rm max})$ as a function of (β, A). The other parameters are kept fixed at $\sigma_\omega = 0.01$, $\sigma_C = 0.01$, and $\sigma^2_W = 0.001$.

[1] D. M. Abrams, R. E. Mirollo, S. H. Strogatz, and D. A. Wiley, Physical Review Letters **101**, 084103 (2008).