Supplementary material for: Rheology of High-Capillary Number Two-Phase Flow in Porous Media

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I. INTERFACE DYNAMICS IN THE NETWORK MODEL

The equations discussed in the article describe how the fluids and the interfaces move inside the links. In addition, we need algorithms to distribute the fluids to the next links and create new interfaces after they mix at the nodes. A set of algorithms for the the interface dynamics can be found in [1], however they were motivated towards the transients. We implement two algorithms for the steady state, each algorithm constitutes a set of *simplified* rules at each time step, as when the steady-state properties are concerned, fine details of dynamics are generally dropped out. We call one algorithm as the bubble controlled (A or B in the main article), where we control the minimum size of a bubble while entering a link. The other is interface controlled (C or D in the main article), where we control the maximum number of interfaces in each link. In the following, we will first describe the interface controlled algorithm and then will highlight the main differences of the bubble controlled algorithm.

A. The interface controlled algorithm:

The algorithm can be divided into three intermediate steps: (a) moving the fluids into each node from the links with incoming flow (interface_move), (b) moving accumulated fluids from each node to the links with outgoing flow and creating new interfaces (interface_create), and (c) controlling the maximum number of interfaces in any link by merging nearest interfaces (interface_merge). We describe these three steps in the following.

(a) interface_move: At a time step, when all the pressures (p_i) at the nodes (i) for an existing fluid configuration are known from the solutions of Kirchhoff equations, a time interval (Δt) is decided. It is decided in such a way that an interface inside any link does not move more than 10% of corresponding link length for the flow rates at that time step. The set of p_is decide which links, among all the links connected to a node, have fluids that flow towards the node and have fluids that flow away from the node at that time step. We name respectively these links as the *incoming links* and the *outgoing links* for that node. Fluid interfaces inside each link j are moved by a distance determined by Δt in the direction of flow and due to this move, a node i receives a certain volume of wetting and/or non-wetting fluids which exit from all the incoming links j and enter node i.

(b) interface_create: All the pore space in this model are assigned only to the links of the network, and the nodes do not contain any real volume. Therefore, the total volume of incoming fluids to a node are to be placed at the beginning of the outgoing links at the same time step. This will create new bubbles and interfaces at the beginning of the outgoing links and the Kirchhoff equations ensure the volume conservation here. Here one needs to decide about the distribution of each individual fluid, that is, how much of the wetting and the non-wetting volumes will enter into an outgoing link j. We calculate this by a democratic rule, that means both the wetting and non-wetting fluids get the same proportion for a link, decided by its flow rate. These democratic rules are symmetric in terms of the wetting and the non-wetting fluids, therefore when the surface tension is zero, we have $\mu_{\text{eff}}(S_{\text{w}}, M) = \mu_{\text{eff}}(S_{\text{n}}, 1/M)$ and $F_{\text{w}} = S_{\text{w}}$. In each link, the wetting and non-wetting bubbles can be placed in two different ways, the wetting first and the non-wetting next, or in the other way. We adopt these two ways alternatively at each consecutive time step. This is one of the main differences of this interface controlled algorithm compared to the bubble-controlled, which we will discuss later.

(c) interface_merge: Placement of new fluid segments (bubbles) inside the outgoing links can increase the number of interfaces indefinitely. We therefore set a maximum limit in the number of interfaces inside any link and introduce a merging rule. After the new interfaces are created, if the total number of interfaces exceeds the maximum number for any link, we merge the two nearest interfaces while conserving the volume of the fluids. The fluid in between the two nearest interfaces are split and added to the next bubbles of the same fluid. We do the splitting in such a way that the center of mass of the two bubbles remains at the same position after merging. We also make sure that, any fluid which is in contact with a node, does not get disconnected during the merging process. Note that, restricting the maximum number of interfaces inside a link does not necessarily impose any restriction on the minimum or maximum

size of a bubble. A complete description of this model will be presented in [2].

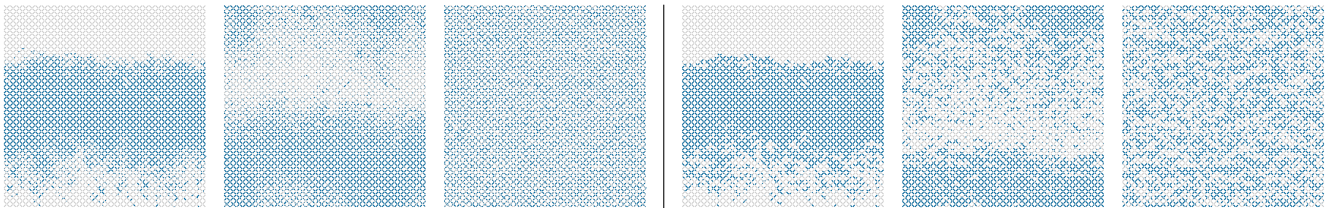
B. The bubble controlled model:

The main difference of this bubble controlled algorithm appears in the order of the two fluids when inserting to an outgoing link. In the interface controlled algorithm, the mixed fluids at a node are distributed with the same proportion to an outgoing link, and it swaps between whether to insert the non-wetting of wetting fluid first or not. The bubble controlled algorithm, on the other hand, also distributes the fluids evenly, but the order in which the fluids are inserted is different. Whether the non-wetting or wetting fluid will be inserted first, depends on the fluid already present in the link. If there is e.g. a non-wetting bubble in the outgoing link in contact with the node that is smaller than a length ($b_{\min}$), we place the wetting fluid in front of the non-wetting fluid. This is motivated by the size of the non-wetting bubble not being large enough to fill up and block the equivalent pore-body. If, on the other hand, the non-wetting bubble is larger than $b_{\min}$, any non-wetting fluid will be inserted first, adding to the bubble already present, and any wetting fluid will be inserted behind it. Bubbles of wetting fluid are treated in the same manner. The value of $b_{\min}$ should, in general, depend on the local pore geometry and on the flow rates, as faster flow rates results in smaller bubbles, described as the drop-traffic flow in [3]. A lower threshold can be used to account the drop-traffic flow, and a large threshold corresponds to high surface tension. For simplicity, we may set $b_{\min}$ proportional to the radius of the link.

In addition, the fluids tend to remain connected due to interfacial tension and not split into very small bubbles. In practice, this is modeled as follows. After the distribution procedure described above, for each node, we identify adjacent wetting and non-wetting bubbles that are smaller than $b_{\min}$. If a node has small adjacent bubbles of both wetting and non-wetting fluid, we reduce the size of all these bubbles, by moving menisci towards the node, to the greatest extent admitted by mass conservation.

All in all, this results in a bubble size distribution in T-junctions as reported in [4]. Note that there is no enforced restriction on the number of bubbles in each link with this approach. In addition, for small enough time steps, the fluid configurations are independent of the time step lengths. The model, with detailed description of the bubble rules, will be presented in [5].

II. FLUID MORPHOLOGIES FROM NETWORK SIMULATIONS



(a) Maximum 4 interfaces. (b) Maximum 2 interfaces.

FIG. 1 Time evolution of fluids for the interface controlled model with a two-dimensional 64×64 square network where the overall flow direction is from the bottom to the top. The wetting and non-wetting fluids are colored by gray and blue respectively. The left three figures correspond to the simulations with maximum 4 interfaces (case C) and the right figures are for maximum 2 interfaces (case D). Here the saturation $S_{\mathrm{w}} = 0.5$ and the viscosity ratio $M = 5$. In each set, the left most figure is after a few time steps, the middle is at some intermediate state and the right most is at the steady state. A significant difference in the two steady-state configurations, corresponding to (a) $\alpha = 1.0$ and (b) $\alpha = 0.6$, can be observed.

The time evolution of the fluids reaching the steady state with the interface controlled model with a two-dimensional 64×64 square network are shown in fig 1 for the maximum (a) 4 and (b) 2 allowed interfaces per link. A noticeable difference between the steady-state fluid distributions for the two cases can be observed. In the steady state for (a), a homogeneous mixing of the two fluids in each link is observed, which leads to $\alpha = 1.0$. On the other hand, links are found to be more saturated with one kind of fluid in (b), which gives rise to $\alpha = 0.6$ in the steady state. Fig 2 shows the snapshots of time evolution for the three dimensional reconstructed Berea network for the interface controlled model with maximum 4 interfaces.

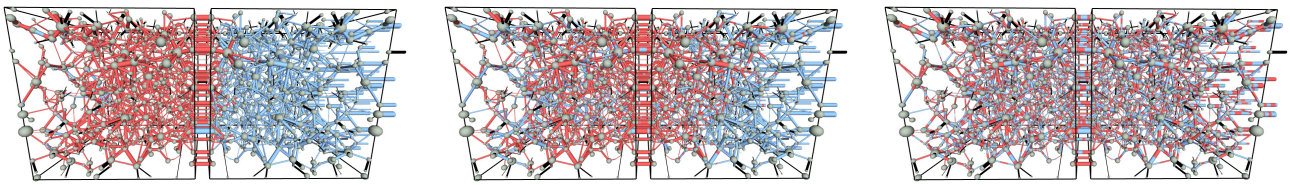


FIG. 2 Simulation snapshots for the three dimensional reconstructed Beria network with the interface controlled model. There are maximum 4 interfaces allowed per link here. Wetting and non-wetting fluids are colored by blue and red respectively. The simulations are with saturation $S_{\rm w} = 0.5$ and viscosity ratio $M = 5$.

III. FRACTIONAL FLOW AT DIFFERENT REGIMES

Figure 3 shows the wetting volumetric fractional flow rate F_w as a function of the wetting saturation S_w for a maximum of 4 (C) or 2 interfaces (D). The data for more interfaces, which correspond to $\alpha = 1$, show $F_w = S_w$ both in 2D and 3D. This indicates that when the maximum number of interfaces in the links is higher, the fluids mix at the link level and flow with the same velocity, so they act as if there is no viscosity contrast between them. On the other hand, when the maximum number of interfaces is less, the viscosity contrast it felt and the least viscous fluids flow the fastest.

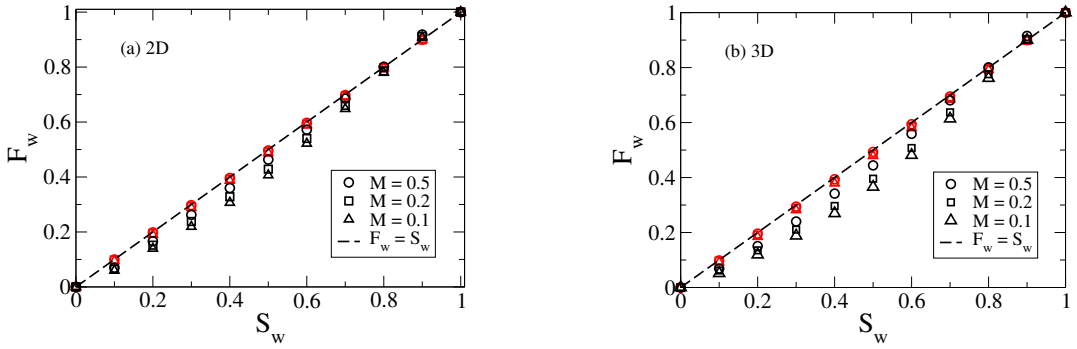
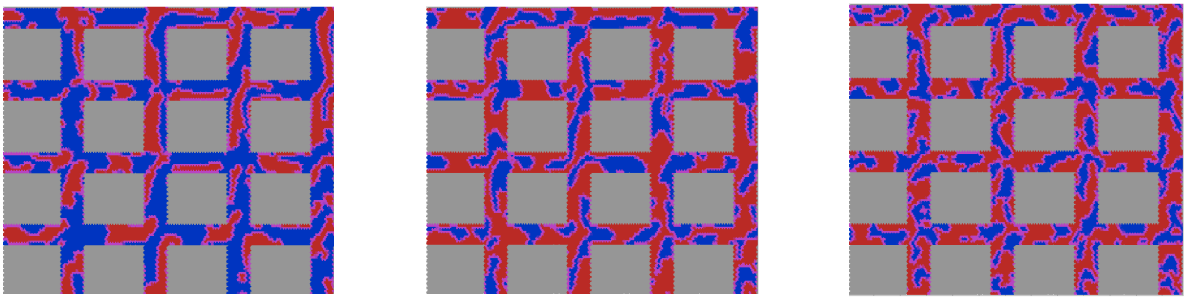


FIG. 3 Comparison of the wetting fractional flow (F_w) for the simulations with many and few interfaces, models C and D respectively. The red symbols are for C and the black symbols are for D. The straight lines represent $F_w = S_w$, the same velocity for both the fluids.

IV. STEADY-STATE FLUID DISTRIBUTION IN LATTICE BOLTZMANN MODEL

Organization of the fluids in the steady state in lattice Boltzmann simulations are shown in fig 4 for the wetting saturations $S_w = 0.4$, 0.6 and 0.8, where string- and plug-like structures can be observed.



(a) $S_w = 0.4$ (b) $S_w = 0.6$ (c) $S_w = 0.8$

FIG. 4 Organizations of the more viscous red (wetting) and less-viscous blue (non-wetting) fluids in the steady state from the lattice Boltzmann simulations for three different saturations. The overall flow is along the from bottom left to the top right direction. A combination of the fluid elements in both the parallel and in series, i.e. both string- and plug-like structures, can be observed.

References

[1] Aker E, Måløy KJ, Hansen A, Batrouni GG. A two-dimensional network simulator for two-phase flow in porous media. *Transp Porous Med.* (1998) **32**:163. doi: 10.1023/A:1006510106194

[2] Sinha S, Vassvik M, Gjennestad MA, Hansen A. In preperation (2019).

[3] Avraam DG, Payatakes AC. Flow regimes and relative permeabilities during steady-state two-phase flow in porous media. *J Fluid Mech.* (1995) **293**:207. doi: 10.1017/S0022112095001698

[4] Garstecki P, Fuerstman MJ, Stonec HA, Whitesides GM. Formation of droplets and bubbles in a microfluidic T-junction-scaling and mechanism of break-up. *Lab Chip.* (2006) **6**:437. doi: 10.1039/b510841a

[5] Vassvik M, Gjennestad MA, Sinha S, Hansen A. In preperation (2019).

[6] Gunstensen AK, Rothman DH, Zaleski S, Zanetti G. Lattice Boltzmann model of immiscible fluids. *Phys Rev A.* (1991) **43**:4320. doi: 10.1103/PhysRevA.43.4320