(Appendix) Micromechanics-based homogenization of the effective physical properties of composites with an anisotropic matrix and interfacial imperfections

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Appendix A: Eshelby tensor (S) for an ellipsoidal shape inclusion in elastostatics for an isotropic medium

For an ellipsoidal inclusion with a symmetric axis (x_1) in an isotropic matrix, the Eshelby tensor can be expressed as shown below,

$$S_{1111}=\frac{1}{2(1-\nu_0)}\left\{1-2\nu_0+\frac{3\rho^2-1}{\rho^2-1}-\left[1-2\nu_0+\frac{3\rho^2}{\rho^2-1}\right]g\right\}$$

$$S_{2222}=S_{3333}=\frac{3}{8(1-\nu_0)}\frac{\rho^2}{\rho^2-1}+\frac{1}{4(1-\nu_0)}\left[1-2\nu_0-\frac{9}{4(\rho^2-1)}\right]g$$

$$S_{2233}=S_{3322}=\frac{1}{4(1-\nu_0)}\left\{\frac{\rho^2}{2(\rho^2-1)}-\left[1-2\nu_0+\frac{3}{4(\rho^2-1)}\right]g\right\}$$

$$S_{2211}=S_{3311}=-\frac{1}{2(1-\nu_0)}\frac{\rho^2}{\rho^2-1}+\frac{1}{4(1-\nu_0)}\left\{\frac{3\rho^2}{\rho^2-1}-(1-2\nu_0)\right\}g$$

$$S_{1122}=S_{1133}=-\frac{1}{2(1-\nu_0)}\left[1-2\nu_0+\frac{1}{\rho^2-1}\right]+\frac{1}{2(1-\nu_0)}\left[1-2\nu_0+\frac{3}{2(\rho^2-1)}\right]g$$

$$S_{2323}=\frac{1}{4(1-\nu_0)}\left\{\frac{\rho^2}{2(\rho^2-1)}+\left[1-2\nu_0-\frac{3}{4(\rho^2-1)}\right]g\right\}$$

$$S_{1212}=S_{1313}=\frac{1}{4(1-\nu_0)}\left\{1-2\nu_0-\frac{\rho^2+1}{\rho^2-1}-\frac{1}{2}\left[1-2\nu_0-\frac{3(\rho^2+1)}{\rho^2-1}\right]g\right\}$$

where ρ is the aspect ratio of the filler $(a_1/a_2=a_1/a_3)$ and ν_0 is the Poisson ratio of the matrix. Other components can be obtained using the minor symmetry condition $(S_{ijkl}=S_{jikl}=S_{ijlk})$. Here, g is given by

$$g \quad =\frac{\rho}{(\rho^2-1)^{3/2}}\left\{\rho(\rho^2-1)^{\frac{1}{2}}-\cosh^{-1}\rho\right\} \qquad \text{prolate shape}$$

$$=\frac{\rho}{(1-\rho^2)^{3/2}}\left\{\cos^{-1}\rho-\rho(1-\rho^2)^{\frac{1}{2}}\right\} \qquad \text{oblate shape}$$

Appendix B: Effective inclusion method

In the effective inclusion method, a displacement jump due to interfacial damage is modeled by the reduced elastic modulus of the inclusion in the perfect bonding case. If this method is adapted, it becomes unnecessary to consider the modified Eshelby tensor or modified strain concentration tensor because interfacial damage is already considered by reducing the elastic modulus of the inclusion.

In the presence of an interfacial spring, the volume-averaged stress$(\overline{\boldsymbol{\sigma}})$ and strain$(\overline{\boldsymbol{\varepsilon}})$ of the composite are expressed using the volume-averaged stress (strain) within the matrix $\boldsymbol{\sigma_0}(\boldsymbol{\varepsilon_0})$ and particles $\boldsymbol{\sigma_1}(\boldsymbol{\varepsilon_1})$, as in Eq. (C1)

$$\begin{aligned} \overline{\boldsymbol{\sigma}} &= \boldsymbol{c_0}\overline{\boldsymbol{\sigma_0}} + \boldsymbol{c_1}\overline{\boldsymbol{\sigma_1}} \\ \overline{\boldsymbol{\varepsilon}} &= c_0\overline{\boldsymbol{\varepsilon_0}} + c_1\overline{\boldsymbol{\varepsilon_1}} + \frac{1}{2V_1}\int_{S_1} (\boldsymbol{\Delta}\mathbf{u} \otimes \boldsymbol{n} + \boldsymbol{n} \otimes \boldsymbol{\Delta}\mathbf{u})dS \end{aligned} \quad \text{(C1)}$$

Here, V_1 and S_1 are the total volume and surface of the particles in a composite, respectively. Under the assumption that local stress at the interface is uniform according to the mean stress of the inclusion, by replacing $\boldsymbol{\Delta u}$ by Eq. (35), Eq. (C1) is reduced and can be expressed by Eq. (C2),

$$\overline{\boldsymbol{\varepsilon}} = c_0\overline{\boldsymbol{\varepsilon_0}} + c_1(\boldsymbol{I} + \boldsymbol{H}{:}\,\boldsymbol{L_1}){:}\,\boldsymbol{L_1^{-1}}{:}\,\overline{\boldsymbol{\sigma_1}} \quad \text{(C2)}$$

where $H_{ijkl} = \frac{1}{4V_1}\int_{S_1}\left(\eta_{ik}n_jn_l + \eta_{il}n_jn_k + \eta_{jk}n_in_l + \eta_{jl}n_in_k\right)dS$

The term $(\boldsymbol{I} + \boldsymbol{H}{:}\,\boldsymbol{L_1}){:}\,\boldsymbol{L_1^{-1}}$ denotes the compliance of the effective inclusion. When the inclusion is spherical with the replacement of $\boldsymbol{\eta}$ by Eq. (34), the reduced bulk and shear modulus $(K_1^R,\ \mu_1^R)$ of the inclusion are obtained as shown below.

$$K_1^R = \frac{K_1 R}{3K_1\beta + R}$$

$$\mu_1^R = \frac{5\mu_1 R}{5R + 2\mu_1(3\alpha + 2\beta)} \tag{C3}$$

After predicting the strain concentration tensor using the reduced elastic stiffness, we can obtain the effective modulus of the composite, which is expressed as Eq. (C4).

$$\boldsymbol{L}_{\mathbf{eff}} = (c_0\boldsymbol{L}_0 + c_1\boldsymbol{L}_1^{\boldsymbol{R}}:\boldsymbol{T}^{\boldsymbol{R}}):(c_0\boldsymbol{I} + c_1\boldsymbol{T}^{\boldsymbol{R}})^{-1}.$$
$$\text{where} \quad \boldsymbol{T}^R = [\boldsymbol{I} + \boldsymbol{S}:\boldsymbol{L}_0:(\boldsymbol{L}_1^R - \boldsymbol{L}_0)]^{-\mathbf{1}} \tag{C4}$$

It should also be noted that when the effective inclusion method is adopted, a perfect bonding condition would be applied at the interface.