Design coefficients

Reinforced concrete slabs, modelled with solid and bar elements, provide stresses, σ, at the nodes, which are computed from the integration points with a stress recovery numerical technique. However, the design of reinforced concrete slabs is based on moments per unit width, so it was necessary to compute them, according to the following equation:

$$M_x = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_x(z)\, z dz \tag{1}$$

where σ_x is the stress function in the *x* direction, *z* is the reference vertical axis and *t* is the slab thickness. In the linear elastic analyses, it was no necessary to use the equation (1) to compute the moments because plate finite elements directly provide them. In the nonlinear analyses, the computation of bending moments with eq. (1) was performed by integrating the stress function along eight elements of the overall thickness, using a linear interpolation as shown in Fig. 1. The stress interpolation function is in agreement with the derivative of the interpolation functions of the nodal displacements.

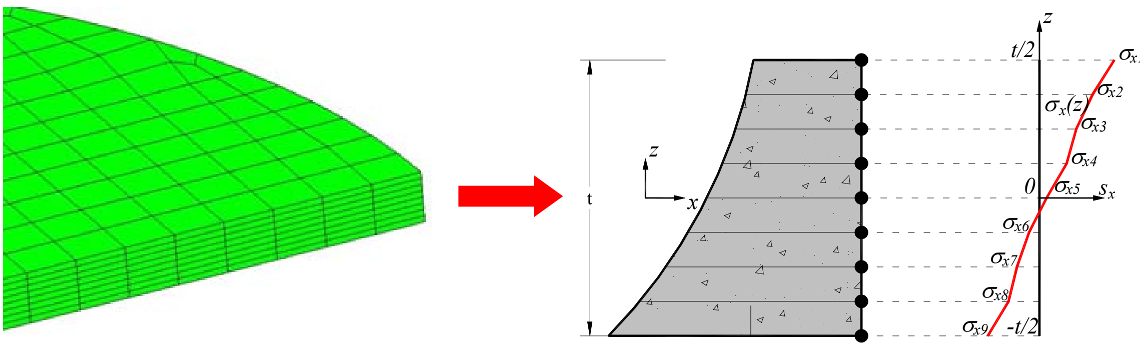

Fig. 1. Stress distribution at the slab along the overall thickness.

Once the moments were obtained, the coefficients of bending moments were computed with the following equation:

$$\alpha = \frac{M}{wl^2} \tag{2}$$

where α is the coefficient, *w* is the load per unit area, *l* is the reference length (*r* , *b* and *h* for circular, elliptic and triangular slabs, respectively) and *M* is the moment computed with the equation (1).

The coefficients of the circular slab were computed at the centre, where the maximum positive moments occurred and at the zones *A* and *B*, where maximum negative moments occurred, according to Fig. 2a. Despite the steel reinforcement is not axisymmetric, it is considered that moment values at any point along the arch *A-B* are almost the same because vertical loading and the model are axisymmetric. In the elliptic slabs, the coefficients were computed at the centre of the slab, where the maximum moments are positive, and at the points *A* and *B*, where the maximum moments are negative, as shown in Fig. 2b. In slab with elliptic geometry, there are coefficients with different magnitudes in each direction at the central zone; at the point *A*, the coefficients are proposed to compute the moments in the horizontal direction, while at the point *B* for computing the moments in the vertical direction. In triangular clamped slabs, shown in Fig. 2c, the *A, B* and *C* zones show negative moments, at the contrary, at the centre zone, positive moments are shown. The moment values in the *C* zone were computed 30° with respect to the horizontal axis, which is parallel to the principal stress direction. In the case of the simple supported slab, the unfavourable zone is the bottom centre because this is where maximum moments occurred.

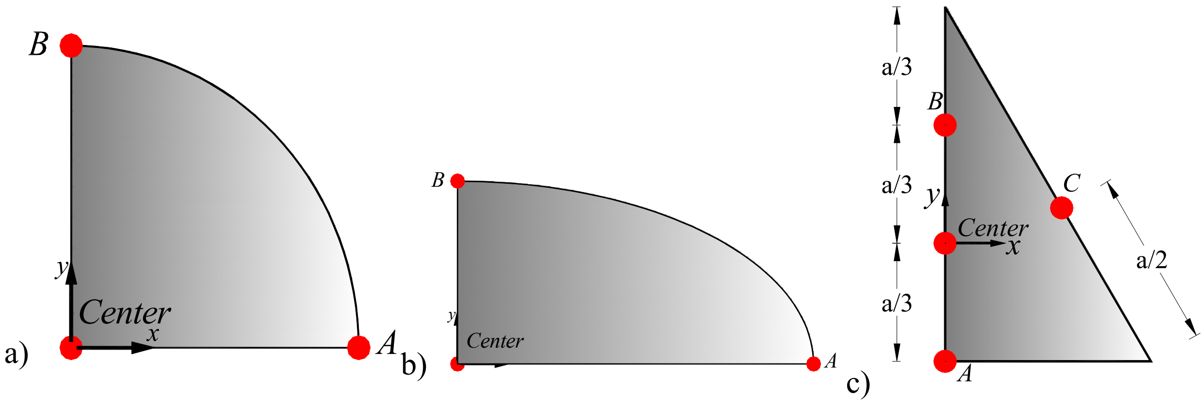

Fig. 2. Moment reference zones in slabs with geometry: a) circular, b) elliptic and c) triangular.

The variation of the computed coefficients in the circular slab under simple supported condition is shown in Fig. 3a, which is identical at any radial direction for its axisymmetric geometry. In the case of the clamped circular slab, the maximum positive moments occur at the centre and the maximum negative moments occur at the edges, as shown in Fig. 3b. For both support conditions, it is observed that when cracking starts, the coefficient magnitudes decrease at the centre and at the edges. Subsequently, the coefficients increase for the stress redistribution in the concrete and yielding in the steel reinforcement. The coefficient magnitudes at the edges are greater than those are in the elastic range. On the contrary, the coefficient magnitudes at the centre are less than those are in the elastic range.

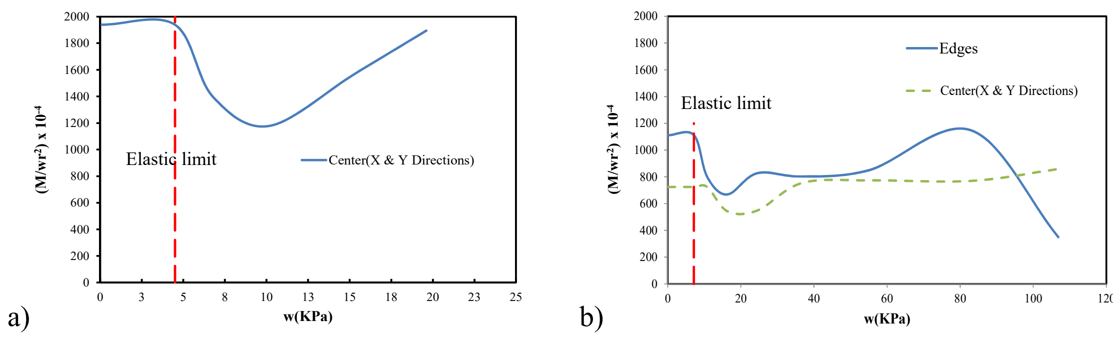

Fig. 3. Coefficient variation *vs*. distributed load in circular slab: a) simple supported and b) clamped.

Figs. 4 to 8 show the variation of the coefficients with ratios of 0.5, 0.6, 0.7, 0.8 and 0.9, respectively. The variation of the coefficients is greater in the y direction under the simple supported condition; this behaviour is observed for both ranges: elastic and nonlinear; however, in the nonlinear range, the coefficients decrease and increase again without reaching the coefficient magnitudes in the elastic range. In the case of clamped slabs, the maximum value is showed at the *B* zone in the elastic range; subsequently, in the nonlinear range, the values of this coefficients shows a variation, particularly those with ratio from 0.5 to 0.7. The other ratios show the maximum moments at the central zone in the x direction for the influence of the stress redistribution by cracking in concrete and yielding of the steel reinforcement.

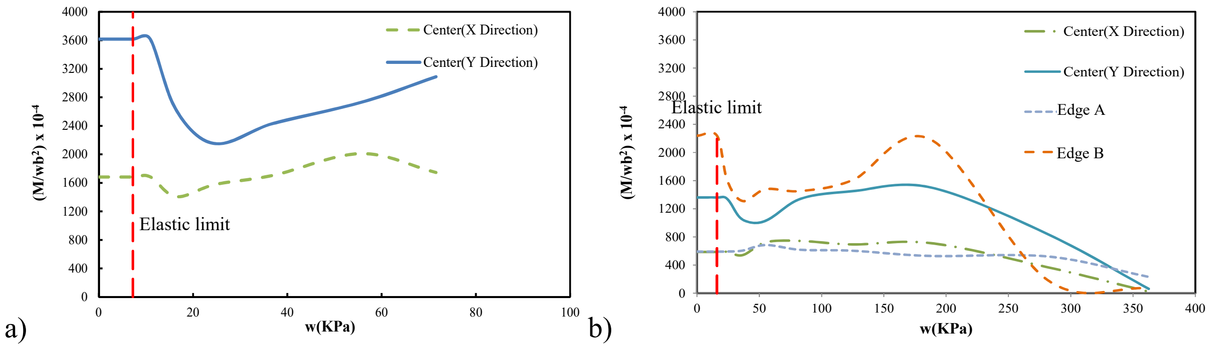

Fig. 4. Coefficient variation *vs*. distributed load in elliptic slab with ratio b/a=0.5: a) simple supported and b) clamped.

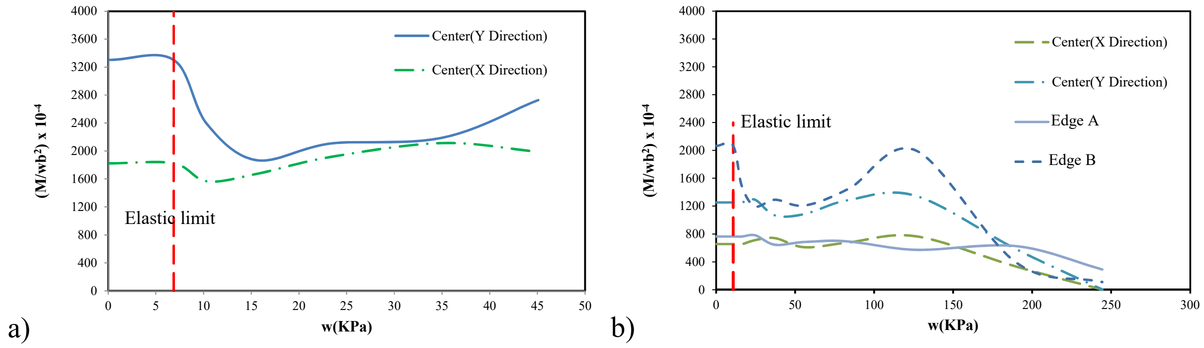

Fig. 5. Coefficient variation *vs*. distributed load in elliptic slab with ratio b/a=0.6 : a) simple supported and b) clamped.

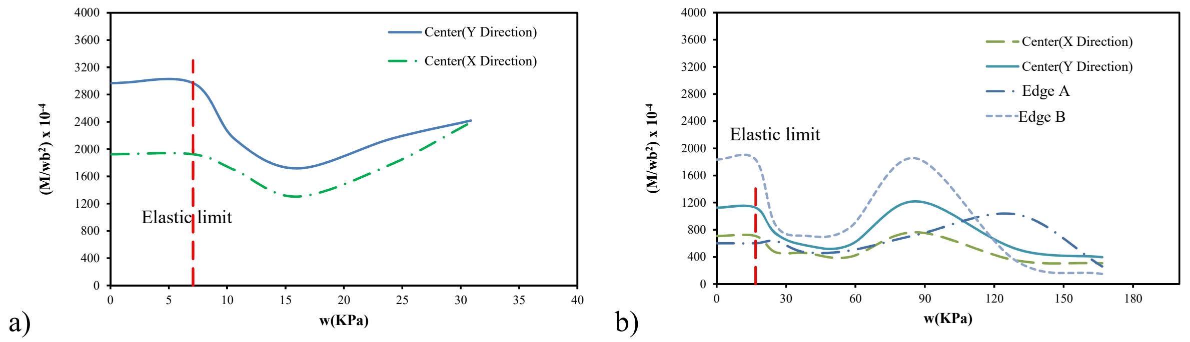

Fig. 6. Coefficient variation *vs*. distributed load in elliptic slab with ratio b/a=0.7 : a) simple supported and b) clamped.

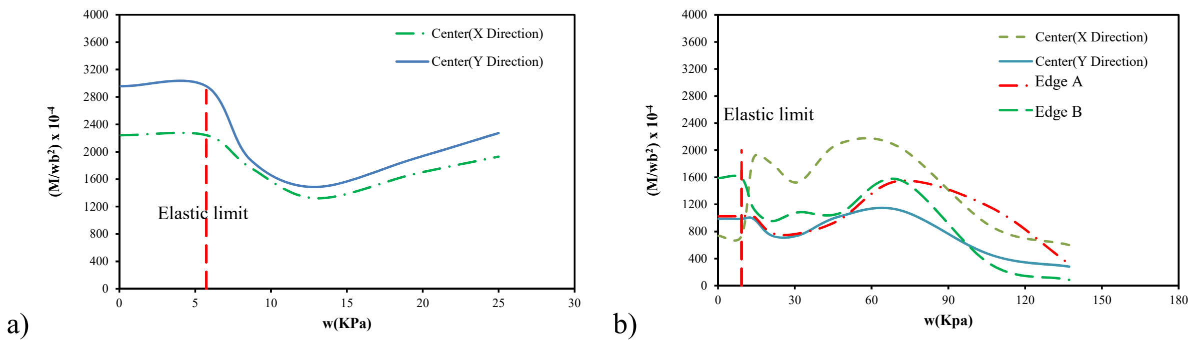

Fig. 7. Coefficient variation *vs*. distributed load in elliptic slab with ratio b/a=0.8: a) simple supported and b) clamped.

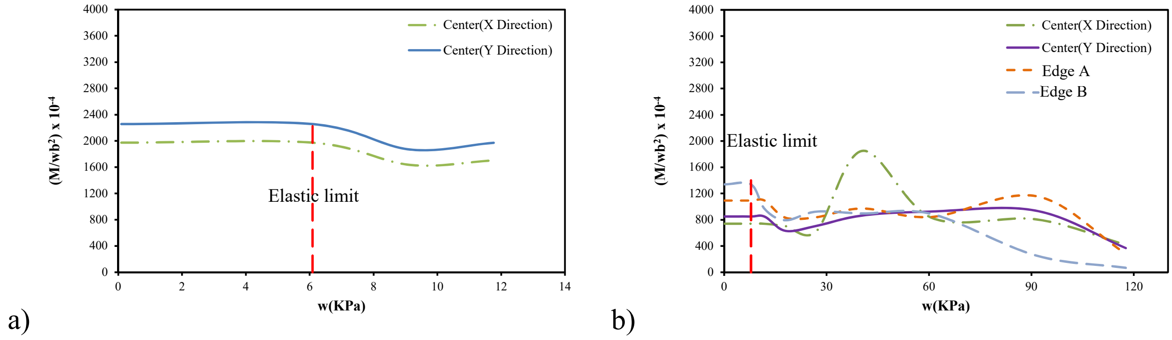

Fig. 8. Coefficient variation *vs*. distributed load in elliptic slab with ratio b/a=0.9: a) simple supported and b) clamped.

The maximum values of the coefficients in the simple supported triangular slab are at the centre on horizontal direction for the elastic range, a condition which changes in the nonlinear range, since the maximum moments occurred at the centre on the vertical direction, as shown in Fig. 9a. In the case of clamped slabs, the maximum value is showed at the *C* zone for both elastic and inelastic ranges. However, at the beginning of the inelastic range, the coefficient magnitudes are reduced. Subsequently, they increase with respect to the elastic range, as shown in Fig. 9b.

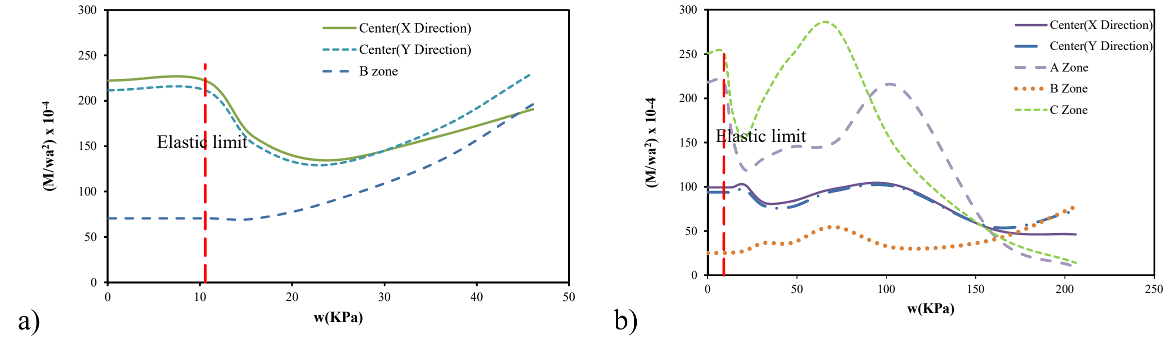

Fig. 9. Coefficient variation *vs*. distributed load in triangular slab: a) simple supported and b) clamped.

In general, the numerically computed coefficients in the nonlinear range show decrement or increment of their magnitudes with respect to those in the elastic range. The variations of these magnitudes are attributed to the occurrence of cracking in concrete and hardening in steel reinforcement in the nonlinear range at the zones with maximum stresses of the slab. In this paper, the coefficient method is proposed for its simplicity to calculate design bending moments in slabs. The magnitudes of the proposed coefficients for designing were taken from the FE models in their linear range. The design coefficients for circular, elliptic and triangular slabs are given in tables 1 to 3, respectively. These design coefficients must be multiplied by $10^{-4}wl^2$ to obtain the flexural design moments per unit width. Two types of construction were considered: case I, slabs built monolithically with their supports and case II, slabs not built monolithically with their supports. In the last type of construction, there are coefficients with only positive values in the centre of slabs, and coefficients with zero values at the edges of the slabs due to supports with zero torsional stiffness.

Table 1. Design coefficients for circular slabs.

Moment	I	II
Neg. at the edges	677	0
Positive	1095	1026

Table 2. Design coefficient for elliptic slabs.

Moment	Span	Span ratios short to long (b/a)									
		0.5		0.6		0.7		0.8		0.9	
		I	II	I	II	I	II	I	II	I	II
Neg. at the edges	*Short*	2238	0	2066	0	1216	0	1589	0	1339	0
	Long	590	0	764	0	601	0	1003	0	1064	0
Positive	*Short*	1359	3617	1254	3304	741	2945	987	2609	850	2174
	Long	585	1682	655	1824	469	1925	735	1977	731	1901

Table 3. Design coefficients for triangular slabs.

Moment	Span	Height (m)	
		4	
		I	II
Neg. at the edges	*Base(A)*	218	0
	Diagonal (C)	251	0
	Peak(B)	26	70
Positive	*Short*	99	222
	Long	94	212