Supplementary Material

**Sample Stan Code for the ORDM**

data{

int Np;

int Ni;

int Nc;

int Ns;

int Y[Np, Ni];

}

parameters{

simplex[Nc] Vc;

real<lower=0> l1\_1;

real<lower=0> l2\_1;

real<lower=0> l3\_1;

real<lower=0> l4\_1;

real<lower=0> l5\_1;

real<lower=0> l6\_1;

real<lower=0> l7\_1;

real<lower=0> l8\_1;

real l1\_01;

real l2\_01;

real l3\_01;

real l4\_01;

real l5\_01;

real l6\_01;

real l7\_01;

real l8\_01;

real<upper=0> l1\_02;

real<upper=0> l2\_02;

real<upper=0> l3\_02;

real<upper=0> l4\_02;

real<upper=0> l5\_02;

real<upper=0> l6\_02;

real<upper=0> l7\_02;

real<upper=0> l8\_02;

real<upper=0> l1\_03;

real<upper=0> l2\_03;

real<upper=0> l3\_03;

real<upper=0> l4\_03;

real<upper=0> l5\_03;

real<upper=0> l6\_03;

real<upper=0> l7\_03;

real<upper=0> l8\_03;

}

transformed parameters{

vector[Ns] PImat[Ni, Nc];

PImat[1,1,1]=0;

PImat[2,1,1]=0;

PImat[3,1,1]=0;

PImat[4,1,1]=0;

PImat[5,1,1]=0;

PImat[6,1,1]=0;

PImat[7,1,1]=0;

PImat[8,1,1]=0;

PImat[1,2,1]=0;

PImat[2,2,1]=0;

PImat[3,2,1]=0;

PImat[4,2,1]=0;

PImat[5,2,1]=0;

PImat[6,2,1]=0;

PImat[7,2,1]=0;

PImat[8,2,1]=0;

PImat[1,3,1]=0;

PImat[2,3,1]=0;

PImat[3,3,1]=0;

PImat[4,3,1]=0;

PImat[5,3,1]=0;

PImat[6,3,1]=0;

PImat[7,3,1]=0;

PImat[8,3,1]=0;

PImat[1,4,1]=0;

PImat[2,4,1]=0;

PImat[3,4,1]=0;

PImat[4,4,1]=0;

PImat[5,4,1]=0;

PImat[6,4,1]=0;

PImat[7,4,1]=0;

PImat[8,4,1]=0;

PImat[1,1,2]=l1\_01;

PImat[2,1,2]=l2\_01;

PImat[3,1,2]=l3\_01;

PImat[4,1,2]=l4\_01;

PImat[5,1,2]=l5\_01;

PImat[6,1,2]=l6\_01;

PImat[7,1,2]=l7\_01;

PImat[8,1,2]=l8\_01;

PImat[1,1,3]=l1\_01+l1\_02;

PImat[2,1,3]=l2\_01+l2\_02;

PImat[3,1,3]=l3\_01+l3\_02;

PImat[4,1,3]=l4\_01+l4\_02;

PImat[5,1,3]=l5\_01+l5\_02;

PImat[6,1,3]=l6\_01+l6\_02;

PImat[7,1,3]=l7\_01+l7\_02;

PImat[8,1,3]=l8\_01+l8\_02;

PImat[1,1,4]=l1\_01+l1\_02+l1\_03;

PImat[2,1,4]=l2\_01+l2\_02+l2\_03;

PImat[3,1,4]=l3\_01+l3\_02+l3\_03;

PImat[4,1,4]=l4\_01+l4\_02+l4\_03;

PImat[5,1,4]=l5\_01+l5\_02+l5\_03;

PImat[6,1,4]=l6\_01+l6\_02+l6\_03;

PImat[7,1,4]=l7\_01+l7\_02+l7\_03;

PImat[8,1,4]=l8\_01+l8\_02+l8\_03;

PImat[1,2,2]=l1\_01+l1\_1;

PImat[2,2,2]=l2\_01+l2\_1;

PImat[3,2,2]=l3\_01+l3\_1;

PImat[4,2,2]=l4\_01+l4\_1;

PImat[5,2,2]=l5\_01;

PImat[6,2,2]=l6\_01;

PImat[7,2,2]=l7\_01;

PImat[8,2,2]=l8\_01;

PImat[1,2,3]=l1\_01+l1\_1+l1\_02+l1\_1;

PImat[2,2,3]=l2\_01+l2\_1+l2\_02+l2\_1;

PImat[3,2,3]=l3\_01+l3\_1+l3\_02+l3\_1;

PImat[4,2,3]=l4\_01+l4\_1+l4\_02+l4\_1;

PImat[5,2,3]=l5\_01+l5\_02;

PImat[6,2,3]=l6\_01+l6\_02;

PImat[7,2,3]=l7\_01+l7\_02;

PImat[8,2,3]=l8\_01+l8\_02;

PImat[1,2,4]=l1\_01+l1\_1+l1\_02+l1\_1+l1\_03+l1\_1;

PImat[2,2,4]=l2\_01+l2\_1+l2\_02+l2\_1+l2\_03+l2\_1;

PImat[3,2,4]=l3\_01+l3\_1+l3\_02+l3\_1+l3\_03+l3\_1;

PImat[4,2,4]=l4\_01+l4\_1+l4\_02+l4\_1+l4\_03+l4\_1;

PImat[5,2,4]=l5\_01+l5\_02+l5\_03;

PImat[6,2,4]=l6\_01+l6\_02+l6\_03;

PImat[7,2,4]=l7\_01+l7\_02+l7\_03;

PImat[8,2,4]=l8\_01+l8\_02+l8\_03;

PImat[1,3,2]=l1\_01;

PImat[2,3,2]=l2\_01;

PImat[3,3,2]=l3\_01;

PImat[4,3,2]=l4\_01;

PImat[5,3,2]=l5\_01+l5\_1;

PImat[6,3,2]=l6\_01+l6\_1;

PImat[7,3,2]=l7\_01+l7\_1;

PImat[8,3,2]=l8\_01+l8\_1;

PImat[1,3,3]=l1\_01+l1\_02;

PImat[2,3,3]=l2\_01+l2\_02;

PImat[3,3,3]=l3\_01+l3\_02;

PImat[4,3,3]=l4\_01+l4\_02;

PImat[5,3,3]=l5\_01+l5\_1+l5\_02+l5\_1;

PImat[6,3,3]=l6\_01+l6\_1+l6\_02+l6\_1;

PImat[7,3,3]=l7\_01+l7\_1+l7\_02+l7\_1;

PImat[8,3,3]=l8\_01+l8\_1+l8\_02+l8\_1;

PImat[1,3,4]=l1\_01+l1\_02+l1\_03;

PImat[2,3,4]=l2\_01+l2\_02+l2\_03;

PImat[3,3,4]=l3\_01+l3\_02+l3\_03;

PImat[4,3,4]=l4\_01+l4\_02+l4\_03;

PImat[5,3,4]=l5\_01+l5\_1+l5\_02+l5\_1+l5\_03+l5\_1;

PImat[6,3,4]=l6\_01+l6\_1+l6\_02+l6\_1+l6\_03+l6\_1;

PImat[7,3,4]=l7\_01+l7\_1+l7\_02+l7\_1+l7\_03+l7\_1;

PImat[8,3,4]=l8\_01+l8\_1+l8\_02+l8\_1+l8\_03+l8\_1;

PImat[1,4,2]=l1\_01+l1\_1;

PImat[2,4,2]=l2\_01+l2\_1;

PImat[3,4,2]=l3\_01+l3\_1;

PImat[4,4,2]=l4\_01+l4\_1;

PImat[5,4,2]=l5\_01+l5\_1;

PImat[6,4,2]=l6\_01+l6\_1;

PImat[7,4,2]=l7\_01+l7\_1;

PImat[8,4,2]=l8\_01+l8\_1;

PImat[1,4,3]=l1\_01+l1\_1+l1\_02+l1\_1;

PImat[2,4,3]=l2\_01+l2\_1+l2\_02+l2\_1;

PImat[3,4,3]=l3\_01+l3\_1+l3\_02+l3\_1;

PImat[4,4,3]=l4\_01+l4\_1+l4\_02+l4\_1;

PImat[5,4,3]=l5\_01+l5\_1+l5\_02+l5\_1;

PImat[6,4,3]=l6\_01+l6\_1+l6\_02+l6\_1;

PImat[7,4,3]=l7\_01+l7\_1+l7\_02+l7\_1;

PImat[8,4,3]=l8\_01+l8\_1+l8\_02+l8\_1;

PImat[1,4,4]=l1\_01+l1\_1+l1\_02+l1\_1+l1\_03+l1\_1;

PImat[2,4,4]=l2\_01+l2\_1+l2\_02+l2\_1+l2\_03+l2\_1;

PImat[3,4,4]=l3\_01+l3\_1+l3\_02+l3\_1+l3\_03+l3\_1;

PImat[4,4,4]=l4\_01+l4\_1+l4\_02+l4\_1+l4\_03+l4\_1;

PImat[5,4,4]=l5\_01+l5\_1+l5\_02+l5\_1+l5\_03+l5\_1;

PImat[6,4,4]=l6\_01+l6\_1+l6\_02+l6\_1+l6\_03+l6\_1;

PImat[7,4,4]=l7\_01+l7\_1+l7\_02+l7\_1+l7\_03+l7\_1;

PImat[8,4,4]=l8\_01+l8\_1+l8\_02+l8\_1+l8\_03+l8\_1;

}

model {

vector[Nc] contributionsC;

vector[Ni] contributionsI;

//Prior

l1\_1~normal(0,20);

l2\_1~normal(0,20);

l3\_1~normal(0,20);

l4\_1~normal(0,20);

l5\_1~normal(0,20);

l6\_1~normal(0,20);

l7\_1~normal(0,20);

l8\_1~normal(0,20);

l1\_01~normal(0,20);

l2\_01~normal(0,20);

l3\_01~normal(0,20);

l4\_01~normal(0,20);

l5\_01~normal(0,20);

l6\_01~normal(0,20);

l7\_01~normal(0,20);

l8\_01~normal(0,20);

l1\_02~normal(0,20);

l2\_02~normal(0,20);

l3\_02~normal(0,20);

l4\_02~normal(0,20);

l5\_02~normal(0,20);

l6\_02~normal(0,20);

l7\_02~normal(0,20);

l8\_02~normal(0,20);

l1\_03~normal(0,20);

l2\_03~normal(0,20);

l3\_03~normal(0,20);

l4\_03~normal(0,20);

l5\_03~normal(0,20);

l6\_03~normal(0,20);

l7\_03~normal(0,20);

l8\_03~normal(0,20);

Vc~dirichlet(rep\_vector(2.0, Nc));

//Likelihood

for (iterp in 1:Np){

for (iterc in 1:Nc){

for (iteri in 1:Ni){

contributionsI[iteri]= categorical\_lpmf(Y[iterp,iteri]+1| softmax(((PImat[iteri,iterc]))));

}

contributionsC[iterc]=log(Vc[iterc])+sum(contributionsI);

}

target+=log\_sum\_exp(contributionsC);

}

}

**Sample Stan Code for the MORDM**

data{

int Np;

int Ni;

int Nc;

int Ns;

int Y[Np, Ni];

}

parameters{

simplex[Nc] Vc;

real<lower=0> l1\_1;

real<lower=0> l2\_1;

real<lower=0> l3\_1;

real<lower=0> l4\_1;

real<lower=0> l5\_1;

real<lower=0> l6\_1;

real<lower=0> l7\_1;

real<lower=0> l8\_1;

real l1\_01;

real l2\_01;

real l3\_01;

real l4\_01;

real l5\_01;

real l6\_01;

real l7\_01;

real l8\_01;

real<upper=0> step1D1;

real<upper=0> step2D1;

real<upper=0> step3D1;

real<upper=0> step1D2;

real<upper=0> step2D2;

real<upper=0> step3D2;

}

transformed parameters{

vector[Ns] PImat[Ni, Nc];

PImat[1,1,1]=0;

PImat[2,1,1]=0;

PImat[3,1,1]=0;

PImat[4,1,1]=0;

PImat[5,1,1]=0;

PImat[6,1,1]=0;

PImat[7,1,1]=0;

PImat[8,1,1]=0;

PImat[1,2,1]=0;

PImat[2,2,1]=0;

PImat[3,2,1]=0;

PImat[4,2,1]=0;

PImat[5,2,1]=0;

PImat[6,2,1]=0;

PImat[7,2,1]=0;

PImat[8,2,1]=0;

PImat[1,3,1]=0;

PImat[2,3,1]=0;

PImat[3,3,1]=0;

PImat[4,3,1]=0;

PImat[5,3,1]=0;

PImat[6,3,1]=0;

PImat[7,3,1]=0;

PImat[8,3,1]=0;

PImat[1,4,1]=0;

PImat[2,4,1]=0;

PImat[3,4,1]=0;

PImat[4,4,1]=0;

PImat[5,4,1]=0;

PImat[6,4,1]=0;

PImat[7,4,1]=0;

PImat[8,4,1]=0;

PImat[1,1,2]=l1\_01+step1D1;

PImat[2,1,2]=l2\_01+step1D1;

PImat[3,1,2]=l3\_01+step1D1;

PImat[4,1,2]=l4\_01+step1D1;

PImat[5,1,2]=l5\_01+step1D2;

PImat[6,1,2]=l6\_01+step1D2;

PImat[7,1,2]=l7\_01+step1D2;

PImat[8,1,2]=l8\_01+step1D2;

PImat[1,1,3]=l1\_01+step1D1+step2D1;

PImat[2,1,3]=l2\_01+step1D1+step2D1;

PImat[3,1,3]=l3\_01+step1D1+step2D1;

PImat[4,1,3]=l4\_01+step1D1+step2D1;

PImat[5,1,3]=l5\_01+step1D2+step2D2;

PImat[6,1,3]=l6\_01+step1D2+step2D2;

PImat[7,1,3]=l7\_01+step1D2+step2D2;

PImat[8,1,3]=l8\_01+step1D2+step2D2;

PImat[1,1,4]=l1\_01+step1D1+step2D1+step3D1;

PImat[2,1,4]=l2\_01+step1D1+step2D1+step3D1;

PImat[3,1,4]=l3\_01+step1D1+step2D1+step3D1;

PImat[4,1,4]=l4\_01+step1D1+step2D1+step3D1;

PImat[5,1,4]=l5\_01+step1D2+step2D2+step3D2;

PImat[6,1,4]=l6\_01+step1D2+step2D2+step3D2;

PImat[7,1,4]=l7\_01+step1D2+step2D2+step3D2;

PImat[8,1,4]=l8\_01+step1D2+step2D2+step3D2;

PImat[1,2,2]=l1\_01+step1D1+l1\_1;

PImat[2,2,2]=l2\_01+step1D1+l2\_1;

PImat[3,2,2]=l3\_01+step1D1+l3\_1;

PImat[4,2,2]=l4\_01+step1D1+l4\_1;

PImat[5,2,2]=l5\_01+step1D2;

PImat[6,2,2]=l6\_01+step1D2;

PImat[7,2,2]=l7\_01+step1D2;

PImat[8,2,2]=l8\_01+step1D2;

PImat[1,2,3]=l1\_01+step1D1+step2D1+l1\_1+l1\_1;

PImat[2,2,3]=l2\_01+step1D1+step2D1+l2\_1+l2\_1;

PImat[3,2,3]=l3\_01+step1D1+step2D1+l3\_1+l3\_1;

PImat[4,2,3]=l4\_01+step1D1+step2D1+l4\_1+l4\_1;

PImat[5,2,3]=l5\_01+step1D2+step2D2;

PImat[6,2,3]=l6\_01+step1D2+step2D2;

PImat[7,2,3]=l7\_01+step1D2+step2D2;

PImat[8,2,3]=l8\_01+step1D2+step2D2;

PImat[1,2,4]=l1\_01+step1D1+step2D1+step3D1+l1\_1+l1\_1+l1\_1;

PImat[2,2,4]=l2\_01+step1D1+step2D1+step3D1+l2\_1+l2\_1+l2\_1;

PImat[3,2,4]=l3\_01+step1D1+step2D1+step3D1+l3\_1+l3\_1+l3\_1;

PImat[4,2,4]=l4\_01+step1D1+step2D1+step3D1+l4\_1+l4\_1+l4\_1;

PImat[5,2,4]=l5\_01+step1D2+step2D2+step3D2;

PImat[6,2,4]=l6\_01+step1D2+step2D2+step3D2;

PImat[7,2,4]=l7\_01+step1D2+step2D2+step3D2;

PImat[8,2,4]=l8\_01+step1D2+step2D2+step3D2;

PImat[1,3,2]=l1\_01+step1D1;

PImat[2,3,2]=l2\_01+step1D1;

PImat[3,3,2]=l3\_01+step1D1;

PImat[4,3,2]=l4\_01+step1D1;

PImat[5,3,2]=l5\_01+step1D2+l5\_1;

PImat[6,3,2]=l6\_01+step1D2+l6\_1;

PImat[7,3,2]=l7\_01+step1D2+l7\_1;

PImat[8,3,2]=l8\_01+step1D2+l8\_1;

PImat[1,3,3]=l1\_01+step1D1+step2D1;

PImat[2,3,3]=l2\_01+step1D1+step2D1;

PImat[3,3,3]=l3\_01+step1D1+step2D1;

PImat[4,3,3]=l4\_01+step1D1+step2D1;

PImat[5,3,3]=l5\_01+step1D2+step2D2+l5\_1+l5\_1;

PImat[6,3,3]=l6\_01+step1D2+step2D2+l6\_1+l6\_1;

PImat[7,3,3]=l7\_01+step1D2+step2D2+l7\_1+l7\_1;

PImat[8,3,3]=l8\_01+step1D2+step2D2+l8\_1+l8\_1;

PImat[1,3,4]=l1\_01+step1D1+step2D1+step3D1;

PImat[2,3,4]=l2\_01+step1D1+step2D1+step3D1;

PImat[3,3,4]=l3\_01+step1D1+step2D1+step3D1;

PImat[4,3,4]=l4\_01+step1D1+step2D1+step3D1;

PImat[5,3,4]=l5\_01+step1D2+step2D2+step3D2+l5\_1+l5\_1+l5\_1;

PImat[6,3,4]=l6\_01+step1D2+step2D2+step3D2+l6\_1+l6\_1+l6\_1;

PImat[7,3,4]=l7\_01+step1D2+step2D2+step3D2+l7\_1+l7\_1+l7\_1;

PImat[8,3,4]=l8\_01+step1D2+step2D2+step3D2+l8\_1+l8\_1+l8\_1;

PImat[1,4,2]=l1\_01+step1D1+l1\_1;

PImat[2,4,2]=l2\_01+step1D1+l2\_1;

PImat[3,4,2]=l3\_01+step1D1+l3\_1;

PImat[4,4,2]=l4\_01+step1D1+l4\_1;

PImat[5,4,2]=l5\_01+step1D2+l5\_1;

PImat[6,4,2]=l6\_01+step1D2+l6\_1;

PImat[7,4,2]=l7\_01+step1D2+l7\_1;

PImat[8,4,2]=l8\_01+step1D2+l8\_1;

PImat[1,4,3]=l1\_01+step1D1+step2D1+l1\_1+l1\_1;

PImat[2,4,3]=l2\_01+step1D1+step2D1+l2\_1+l2\_1;

PImat[3,4,3]=l3\_01+step1D1+step2D1+l3\_1+l3\_1;

PImat[4,4,3]=l4\_01+step1D1+step2D1+l4\_1+l4\_1;

PImat[5,4,3]=l5\_01+step1D2+step2D2+l5\_1+l5\_1;

PImat[6,4,3]=l6\_01+step1D2+step2D2+l6\_1+l6\_1;

PImat[7,4,3]=l7\_01+step1D2+step2D2+l7\_1+l7\_1;

PImat[8,4,3]=l8\_01+step1D2+step2D2+l8\_1+l8\_1;

PImat[1,4,4]=l1\_01+step1D1+step2D1+step3D1+l1\_1+l1\_1+l1\_1;

PImat[2,4,4]=l2\_01+step1D1+step2D1+step3D1+l2\_1+l2\_1+l2\_1;

PImat[3,4,4]=l3\_01+step1D1+step2D1+step3D1+l3\_1+l3\_1+l3\_1;

PImat[4,4,4]=l4\_01+step1D1+step2D1+step3D1+l4\_1+l4\_1+l4\_1;

PImat[5,4,4]=l5\_01+step1D2+step2D2+step3D2+l5\_1+l5\_1+l5\_1;

PImat[6,4,4]=l6\_01+step1D2+step2D2+step3D2+l6\_1+l6\_1+l6\_1;

PImat[7,4,4]=l7\_01+step1D2+step2D2+step3D2+l7\_1+l7\_1+l7\_1;

PImat[8,4,4]=l8\_01+step1D2+step2D2+step3D2+l8\_1+l8\_1+l8\_1;

}

model {

vector[Nc] contributionsC;

vector[Ni] contributionsI;

//Prior

l1\_1~normal(0,20);

l2\_1~normal(0,20);

l3\_1~normal(0,20);

l4\_1~normal(0,20);

l5\_1~normal(0,20);

l6\_1~normal(0,20);

l7\_1~normal(0,20);

l8\_1~normal(0,20);

l1\_01~normal(0,20);

l2\_01~normal(0,20);

l3\_01~normal(0,20);

l4\_01~normal(0,20);

l5\_01~normal(0,20);

l6\_01~normal(0,20);

l7\_01~normal(0,20);

l8\_01~normal(0,20);

step1D1~normal(0,20);

step2D1~normal(0,20);

step3D1~normal(0,20);

step1D2~normal(0,20);

step2D2~normal(0,20);

step3D2~normal(0,20);

Vc~dirichlet(rep\_vector(2.0, Nc));

//Likelihood

for (iterp in 1:Np){

for (iterc in 1:Nc){

for (iteri in 1:Ni){

contributionsI[iteri]= categorical\_lpmf(Y[iterp,iteri]+1| softmax(((PImat[iteri,iterc]))));

}

contributionsC[iterc]=log(Vc[iterc])+sum(contributionsI);

}

target+=log\_sum\_exp(contributionsC);

}

}