Supplementary document to:

"Ecological validity of impulsive choice: consequences of profitability-based short-sighted evaluation in the producer-scrounger resource competition" by Ogura, Amita and Matsushima (2018) *Frontiers in Applied Mathematics and Statistics* (doi: 10.3389/fams.2018.00049)

Supplementary analysis of an alternative model

Alternatively, we may assume that the food patch F supplies food at a constant food supply rate (s). Supplementary Fig.1S schematically illustrates the alternative models. Fig.2S shows the parameters (A/D ratio of SS and LL) where SS is more profitable than LL for each of the cases shown in Fig.1S.

Profitability is given as follows:

If Φ follows Ω, $$prof_P = \frac{A}{D + {}^{A}/_{s}} \quad \text{(1S)}$$

If Φ precedes Ω, $$prof_P = \frac{A + sT}{2\left(D + {}^{A}/_{s}\right)} \quad \text{(2S)}$$

We will show conditions where

$$prof_p\ (SS) > prof_p\ (LL) \quad \text{(3S)}$$

holds true for each of the following cases.

Case (1); no scrounging for both SS and LL

$$A_{SS} < A_{LL} \leq sT$$

Case (2); scrounging for LL but not for SS

$$A_{SS} \leq sT < A_{LL}$$

Case (3); scrounging for both LL and SS

$$sT < A_{SS} < A_{LL}$$

Case (1)

The inequality (3S) is given in the same way as the original mode, such as

$$\frac{A_{SS}}{D_{SS} + A_{SS}/s} > \frac{A_{LL}}{D_{LL} + A_{LL}/s} \quad \text{(4S)}$$

which is equivalent with a much simpler form for $\forall s > 0$

$$\frac{A_{SS}}{D_{SS}} > \frac{A_{LL}}{D_{LL}} \quad \text{(4'S)}$$

meaning that SS is more profitable than LL, when and only when the A/D ratio is larger for SS than for LL (Figure 2S left).

Case (2)

The inequality formula (3S) is given as

$$\frac{A_{SS}}{D_{SS}+A_{SS}/s} > \frac{A_{LL}+sT}{2(D_{LL}+A_{LL}/s)} \quad \text{(5S)}$$

which is equivalent with

$$f = \frac{s(2D_{LL}A_{SS}-D_{SS}A_{LL})+A_{LL}A_{SS}}{s(sD_{SS}+A_{SS})} > T \quad \text{(5'S)}$$

meaning that *T* has an upper limit given by *f*. As the denominator is positive, an upper limit exists for $\forall s > 0$ if coefficient of *s* in the numerator is larger than 0, namely when

$$\frac{2A_{SS}}{D_{SS}} > \frac{A_{LL}}{D_{LL}} \quad \text{(6S)}$$

holds. If otherwise, the upper limit of *T* exists when food supply rate *s* is above a certain level given by;

$$s > \frac{A_{LL}A_{SS}}{A_{LL}D_{SS}-2A_{SS}D_{LL}} > 0 \quad \text{(7S)}$$

This means that *SS* is more profitable than *LL*, if *T* is short enough, and the upper limit appears whenever the *A/D* ratio of *SS* is higher than the half of *LL* (as (6S) indicates). Even when the *A/D* ratio of *SS* is even lower, a positive *T* value exists so that *SS* is more profitable than *LL*, if *s* is high above the lower limit given by (7S). Similarly to the case (1), *A*/*D* ratios that satisfy the inequality (6S) are illustrated in intermediate brown in Figure 2S center. The area where (7S) matters is illustrated in thin brown in this figure.

Case (3)

The inequality formula (3S) is given as

$$\frac{A_{SS}+sT}{2(D_{SS}+A_{SS}/s)} > \frac{A_{LL}+sT}{2(D_{LL}+A_{LL}/s)} \quad \text{(8S)}$$

which is equivalent with

$$g = \frac{A_{LL}D_{SS}-A_{SS}D_{LL}}{(D_{LL}-D_{SS})s+(A_{LL}-A_{SS})} < T \quad \text{(8'S)}$$

meaning that *T* has a lower limit given by g, when it is positive. If otherwise, and g is 0 or negative, the inequality (9) holds for $\forall T > 0$, namely;

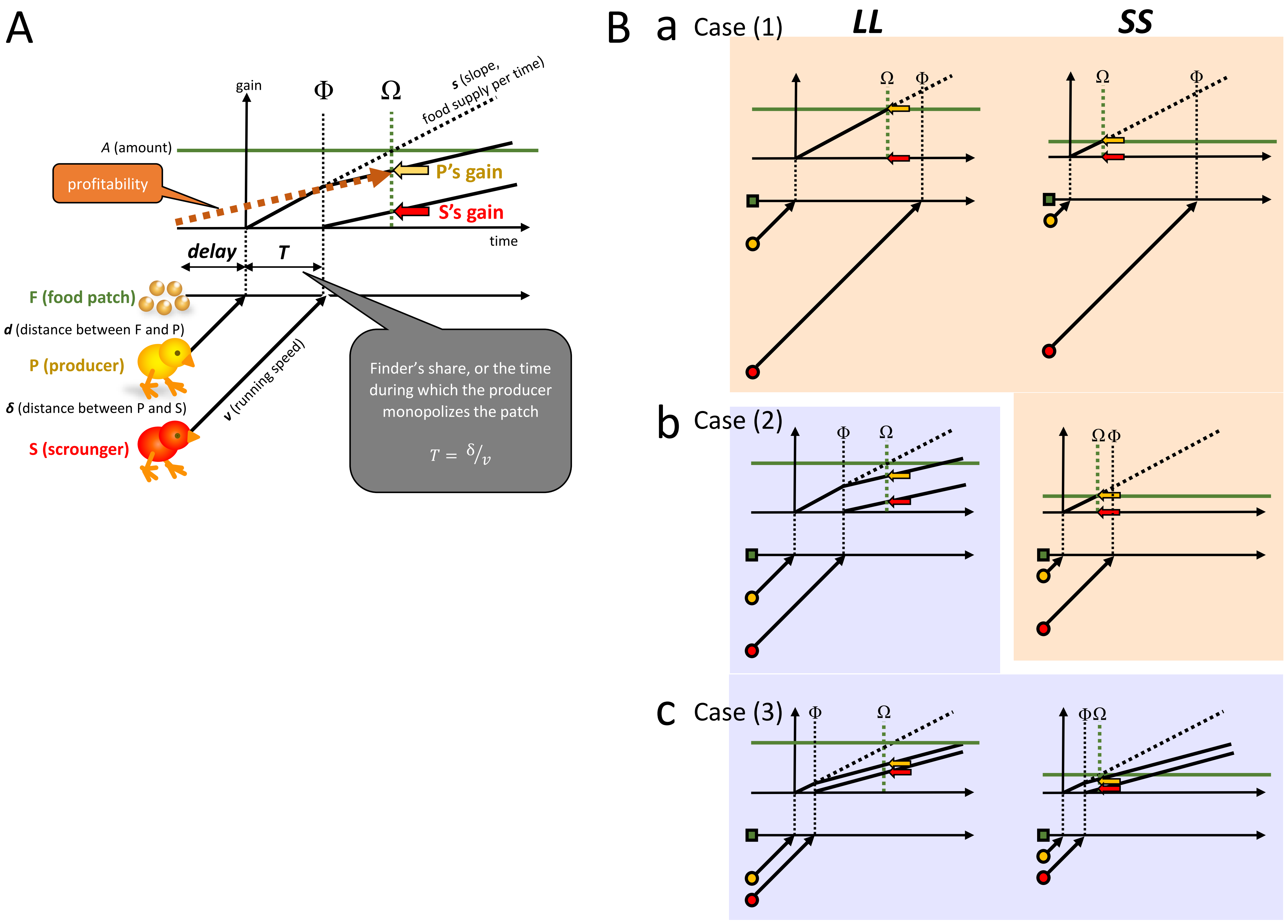
$$\frac{A_{SS}}{D_{SS}} \geq \frac{A_{LL}}{D_{LL}} \quad \text{(9S)}$$

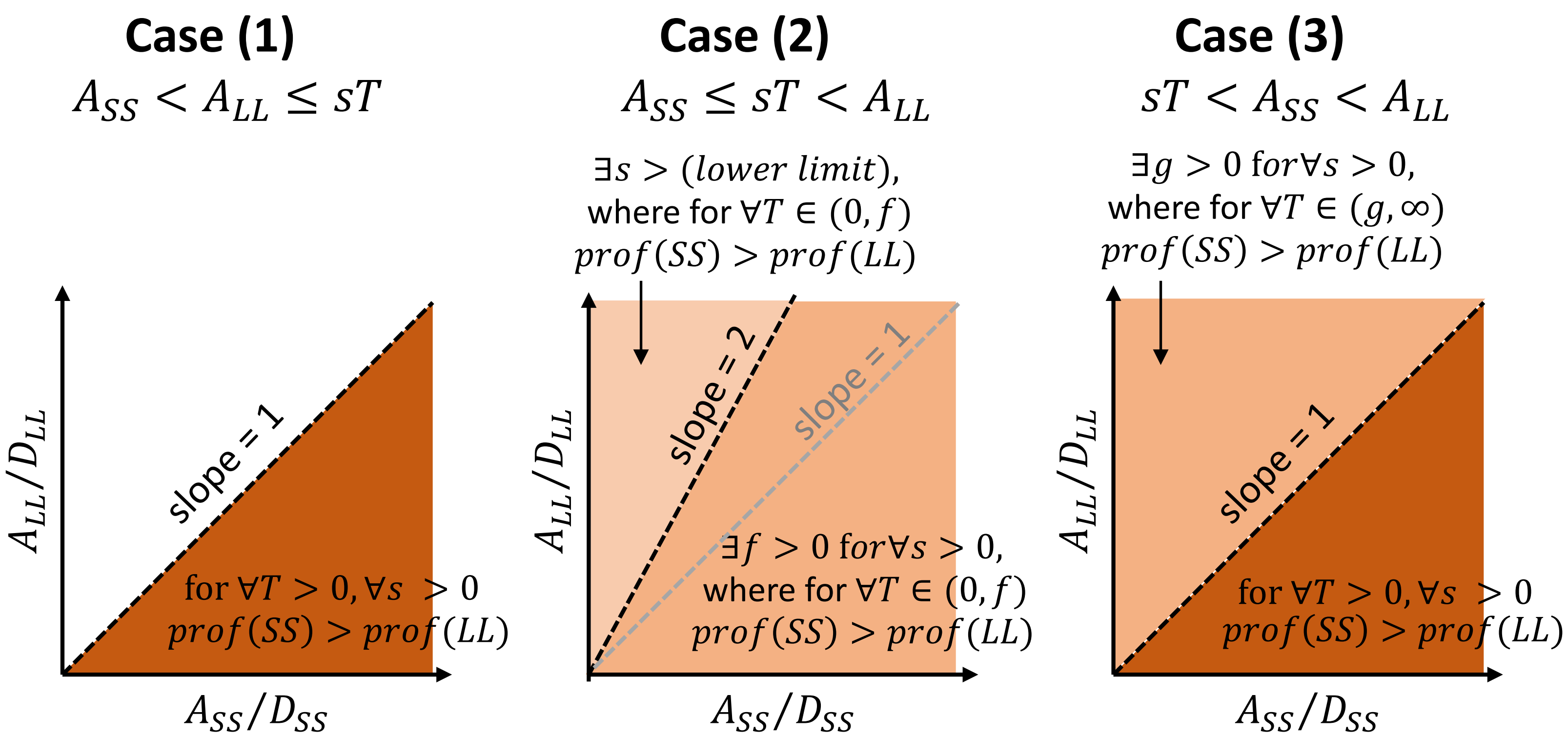
holds. Therefore, *SS* is unconditionally (*i.e.*, for $\forall T > 0$ and $\forall s > 0$) more profitable than *LL*, in the area illustrated in dark brown in Figure 2S right. Even when (9S) does not hold, *SS* can still be more profitable than *LL* if *T* is long enough above g (8'S).

FIGURE 1S Assumptions of the alternative model, in which *s* represents the slope of food supply per time, rather than the speed of food consumption per time per individual. For other terminologies see Figure 2 of the main text.

FIGURE 2S Areas of parameters (*A*/*D* ratio) for *SS* (abscissa) and *LL* (ordinate) where *SS* gives rise to a higher profitability than *LL*, for Case (1), Case (2) and Case (3), respectively.

Ogura et al. Supplementary Fig.1S




Case (1)
$A_{SS} < A_{LL} \leq sT$
slope = 1
for $\forall T > 0, \forall s > 0$
$prof(SS) > prof(LL)$
A_{LL}/D_{LL}
A_{SS}/D_{SS}
Case (2)
$A_{SS} \leq sT < A_{LL}$
$\exists s > (lower\ limit)$,
where for $\forall T \in (0, f)$
$prof(SS) > prof(LL)$
slope = 2
slope = 1
$\exists f > 0$ for $\forall s > 0$,
where for $\forall T \in (0, f)$
$prof(SS) > prof(LL)$
A_{LL}/D_{LL}
A_{SS}/D_{SS}
Case (3)
$sT < A_{SS} < A_{LL}$
$\exists g > 0$ for $\forall s > 0$,
where for $\forall T \in (g, \infty)$
$prof(SS) > prof(LL)$
slope = 1
for $\forall T > 0, \forall s > 0$
$prof(SS) > prof(LL)$
A_{LL}/D_{LL}
A_{SS}/D_{SS}